DOCUMENT RESUME

ED 473 099 SE 067 236

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TITLE	Reshaping Mathematics for Understanding Motion Geometry.
INSTITUTION	Hawaii Univ., Honolulu. Curriculum Research and Development Group.
ISBN	ISBN-1-58351-024-9
PUB DATE	2003-00-00
NOTE	121p.
AVAILABLE FROM	Reshaping Mathematics for Understanding Project, Curriculum Research and Development Group, 1776 University Avenue, Honolulu, HI 96822-2463. e-mail: crdg@hawaii.edu; Web site: http://www.hawaii.edu/crdg.
PUB TYPE	Books (010) -- Guides - Classroom - Teacher (052)
EDRS PRICE	EDRS Price MF01 Plus Postage. PC Not Available from EDRS
DESCRIPTORS	*Geometry; Learning Strategies; *Mathematics Education; Middle Schools; Problem Solving; Teaching Methods; Thinking Skills

ABSTRACT

This book introduces concepts of geometry that students use throughout middle-grade and higher-level mathematics courses. These concepts, presented through the study of transformations, provide a framework for other important topics such as number, measurement, proportional reasoning, and graphing on the coordinate plane. The book is designed for students to learn about the properties of motion and how motion affects objects as they learn about reflections, translations, and rotations. The problems featured in each unit are designed to help students manipulate drawings physically, be accurate in their work, and use precise language in analyzing the results of the motions. (KHR)

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Reshaping Mathematics for Understanding

Motion Geometry

Hannah Slovin
Linda Venenciano
Melanie Ishihara
Cynthia Beppu

Figure C

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Reshaping Mathematics for Understanding

Motion Geometry

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ISBN 1-58351-024-9

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Printed in the United States of America

08 07 06 05 04 03 9 8 7 6 5 4 3 2 1

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Acknowledgments

This undertaking would not have been possible without the tireless support of our student helpers Erin Sakihara, Gilles Martinez, Tricia Saito, and Lindsay Miura. Their patience and enthusiasm helped us hold on to our sanity.

Thanks to the generous support from the Curriculum Research & Development Group's Math Section, whose members have included Gary Martin, Barbara Dougherty, Annette Matsumoto, Sid Rachlin, Claire Okazaki, Fay Zenigami, Mary Pat Sjostrom, Jenny Simmons, Gail Hunger, and David Lani.

We want to express our sincere appreciation to the following teachers for testing some of our material in their classrooms and providing us with feedback: Jean Hamai, Phaedra Link, Elizabeth Dudley-Damon, Dexsilyn Navarro, Emma Smith, and Lynele Nayal. Their comments and suggestions have contributed significantly to the project.

We thank the classroom teachers and others who have attended our professional development institutes for encouraging us to publish these materials (formerly the Reshaping Mathematics Program) and make them available for teacher development workshops and classroom use.

We also thank the generations of University Laboratory School sixth- and seventh-graders who, through interviews, class discussions, and journal writings, taught us so much about how they learned mathematics.

Lastly, mahalo nui loa (thank you) to Dr. Anthony Picard of the University of Hawai'i, whose simple question, "Why not start with geometry?" planted the seed for creating a mathematics course for middle-grades students using a geometric perspective.

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About the series . . .

Reshaping Mathematics for Understanding is a series of fourteen units suitable for sixth- through eighth-grade students that addresses important topics in middle-grades mathematics, including geometry, measurement, proportional reasoning, rational numbers, probability and statistics, and algebra. The entire series is designed to help students learn to think mathematically. It focuses on developing students' understanding of mathematical concepts and on their ability to draw connections among these concepts. It may serve either as the primary resource in the mathematics curriculum or as a complement to other material. The titles for the full series are listed below.

- Getting Started
- Motion Geometry
- Measurement
- Polygons
- Dilations
- Fractions
- Decimals
- Ratio and Proportion
- Area of Polygons
- Solids
- Probability and Statistics
- Integers
- Algebra Patterns and Relationships
- Number Theory

Access for Every Student

The problems and tasks in the *Reshaping Mathematics for Understanding* series are designed to enable every student to approach mathematics through familiar contexts. Building problem situations on students' past experience makes the study of mathematics more accessible and allows them to expand their thinking. Similarly, to promote genuine engagement, many problems have more than one solution path, giving students opportunities to choose the strategies they prefer using to solve problems. Additionally, to give students opportunities to interact with each concept at varying levels of abstractness and generality, the lessons present new ideas over several days. This design feature also allows students to learn at different rates.

Learning with Understanding

Students can learn only what they truly understand. To learn, they must understand the concepts that underlie operations; they must make connections among processes and concepts; and they must know when and how to apply concepts and operations to problems. Understanding mathematics means doing more than learning how to perform calculations to get correct answers. The topics in this series, developed through problems and lab explorations, encourage students to deepen their conceptual understanding through the practice of reasoning and problem solving.

Role of Visual/Spatial Thinking and Reasoning

Mathematics can and should be a lively course of study for students. It should engage them in active inquiry and give them many opportunities to explore problems whose solutions add to their understanding of the world. For many students, however, mathematics involves merely manipulating numbers. With this limited view, they do not learn how to use multiple contexts, tools, and strategies to solve problems or how to integrate mathematical concepts into broader contexts. Transformational (motion) geometry, which plays an important role in the sequence of the concepts in this series, emphasizes the use of visual contexts and spatial thinking. Lesson discussions further encourage students to use their understanding of spatial relations to make connections among concepts. The Motion Geometry and Dilations units, in particular, give students valuable experience with transformations and enhance the study of many related topics.

Unit Design

Each unit in the *Reshaping Mathematics for Understanding* series can be used individually, or units can be used in clusters. Throughout the series, references among units direct teachers and students to tasks that will help them connect their understanding of new concepts to related experiences.

The lessons in each unit are uniquely designed to enable students to progress through a sequence of tasks that maximize learning with understanding. Rather than present a topic in its entirety in one day, the lessons develop concepts over time. Most concepts begin with an open-ended problem that draws on students' previous experience and intuition and allows for multiple responses. The variety of solutions students propose to such a problem helps the teacher assess what background knowledge students bring to the topic. The problems and tasks that follow develop the concept through a sequence of approaches that provide several direct examples of the concept, raise questions for clarification, offer alternative viewpoints, and prompt students to summarize ideas.

The Lessons

There are two forms of lessons in each unit, problem sets and in-class labs. Problem sets consisting of three to five problems are designed to cover several concepts related to the unit topic. Problem sets should be assigned for homework and discussed in class the next day. When students find a problem too difficult to solve on their own, they should be instructed to write questions to help them solve it, and to ask the questions in the discussion. Students work on the labs in small groups in class and debrief afterward, giving them an opportunity to work collaboratively and to concentrate on one strand.

The class discussions are essential in helping students build conceptual understanding. Sharing their solutions and questions allows students to reflect on their thinking and to consider input from others. Both the teacher and students share the responsibility for making discussions productive. As they solve the problems and complete the lab tasks, students explain their thinking, offer alternative responses, and ask questions. Teachers facilitate, asking strategic questions to focus students' thinking on critical ideas. In guiding the discussion, teachers should ensure engagement in the learning process by encouraging students to monitor their learning and by providing a safe, open learning environment in which to share, discuss, and address misconceptions.

Assessment

Except for the Getting Started unit, designed to orient students to the series, all units include suggested assessment items. To check periodically that students understand and that they are participating in the class discussions, teachers can also create "instant" quizzes to give the day after a problem set discussion or lab debriefing. These quizzes should be unannounced and should take between five and ten minutes at the start of class. They have three purposes:

to emphasize the importance of student discussions;

to emphasize the value of information shared by students;

to highlight the importance of asking questions to clarify understanding.

In creating an instant quiz, teachers should focus on students' ideas from the discussion and write two or three short-answer questions as follows:

- a question that is content-based or refers to an understanding that has been established.

  Example: What did we decide it meant to measure the perimeter of a polygon?

- a question that refers to someone's method for solving a problem or alternative point of view.

  Example: What method did Leslie use to find the area of the trapezoid?

- a question that highlights an undefined assumption or a statement that needed clarification.

  Example: What did Jose mean by *proportional*?

Materials

A list of the materials needed for each lesson appears at the front of every unit. Although students need no special materials or equipment to complete the homework problems, it would be helpful to have them available during discussions. Encouraging students to use a broad range of tools to explore and express their thinking promotes greater understanding.

About the unit . . .

The Motion Geometry unit introduces students to concepts of geometry they will use throughout middle-grade and higher-level mathematics courses. These concepts, presented through the study of transformations, provide a framework for other important topics such as number, measurement, proportional reasoning, and graphing on the coordinate plane. As students learn about reflections, translations, and rotations (flips, slides, and turns), they are also learning about the properties of the motions and how they affect objects. The problems in this unit require students to manipulate drawings physically, to be accurate in their work, and to use precise language in analyzing the results of the motions.

Geometric Thinking

Motion geometry in the middle grades can provide a powerful context for students to develop their reasoning processes. Problems are carefully designed to help them progress from using the informal visual and descriptive bases for thinking about geometric objects which are suitable for elementary grades, to using more formal reasoning based on the logical arguments and justifications that are required in advanced courses.

Motion is a part of everyone's daily life—riding a Ferris wheel, sliding into home plate, or flipping a coin. The study of motions—reflections, translations, and rotations—allows students to draw on their experience and their intuition as they strive to understand the physical world. For example, they can compare two triangles by flipping one over to match the angles, sliding one on top of the other to check the areas, or turning one around to compare its shape to another's. These "common sense" methods, which are grounded in mathematical principles, help students to reason and to justify the relationships between the triangles.

The Language of Motion Geometry

Although many of the ideas from motion geometry relate to familiar experiences, we use special terminology to name and describe transformations. Some of the terminology can be confusing to students. For example, the term translation means something different in a foreign language class than it does in a mathematics class. The problems in this unit help students build on their previous conceptions to learn new mathematics terminology, and the suggested discussion questions encourage them to clarify their understanding and to re-define terms in a mathematics context.

Recording Motions on Paper

Solving problems in motion geometry requires students to manipulate drawings physically using pencil, paper, and other tools. They must trace given figures with a reasonable degree of accuracy, use lines and arrows in particular ways, and label drawings effectively to communicate to others how they solved a

problem. Some middle-grades students have difficulty achieving this level of precision. The teacher and students together should determine appropriate expectations for neatness. However, it is important that all students cultivate habits of mind that promote care and mathematical correctness. At the same time, class discussions should include opportunities for students to share and practice techniques for making drawings accurate, and should highlight how the techniques and procedures reflect the mathematical properties of the motions. In this way, the study of the physical and the conceptual components of transformations complement each other. (See page 107 for suggested drawing methods.)

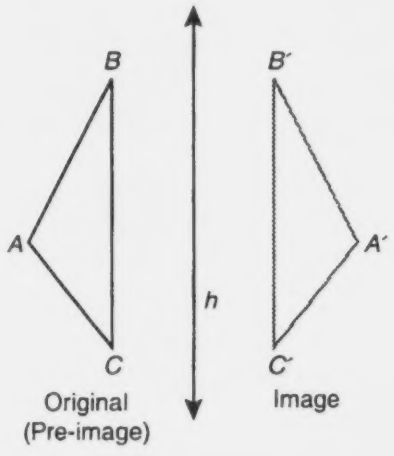
The Strands in Motion Geometry

Introduction to Motion: Students begin by using real objects to explore motion. They experiment with different types of motions, informally describing the properties of the motions they perform. They compare an object before and after the motion; for example, they may focus on its orientation. As they observe that the size, shape, and other physical properties of the objects do not change with rigid motions, students develop a sense of congruence.

Representation: Because an actual motion cannot be preserved on paper, we must use representation to convey how an object should be or has been moved. The early tasks in this unit give students opportunities to invent ways to represent the motions they perform and to explain their drawings. These experiences help students understand the importance of using a common scheme to represent motions and to make sense of some conventional schemes.

The problems in this unit use a set of "tools" to represent how one should move an object or to convey how an object has been moved. These tools are the line of reflection, the translation arrow, and the rotation arrow and center of rotation. Each is discussed in its related strand section.

Reflection: Most students have had some experience with reflections, which is the easiest motion to perform. The sequence of problems in the reflection strand requires students to reflect figures, find lines of reflection, and locate the position of objects before they were reflected. Reflections are performed over a line of reflection. Students may devise different techniques for doing the reflection, but they must follow all conditions as presented in the problem. For example, they cannot move a given line of reflection to a more convenient location on their paper. Students should be able to draw with accuracy both figures and the line used to create the reflection.



The original triangle (in black) is reflected *over* line *h* to get the image (in gray).

Students may see the connection between their experience of reflections and solving problems involving line symmetry, graphing parabolas, or classifying quadrilaterals.

Translation: Translations move a figure a fixed distance in a given direction. Translation arrows (vectors) represent the distance and direction for moving the figure. The problems in this strand give students experience in translating figures, locating figures before they were translated, and suggesting arrows that might have been used in a particular translation. Students need to practice using the translation arrow "tool," which helps them make their drawings accurate. They are expected to draw translation images and arrows accurately and to label the figures correctly.

Students who understand translations can apply this experience to measurement topics—length, area, and volume—to parallel lines, and to graphing on the coordinate plane.

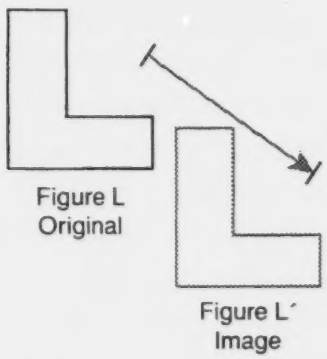


Figure L and its translated image.

Rotation: A rotation is a transformation that turns a figure around one point (or, to use the mathematical term, "*rotates* a figure *about* a point") called the center of rotation. Students learn about the tools used to rotate objects and practice doing rotations. They use degrees and fractions to describe rotations.

Rotations are the most challenging motion for students to do. They may not have had enough experience with circles or be sufficiently adept at using a compass to make precise drawings for problems, that, for example, ask them to locate a center of rotation. They should, however, be able to approximate where a center of rotation is and to sketch possible rotation arrows.

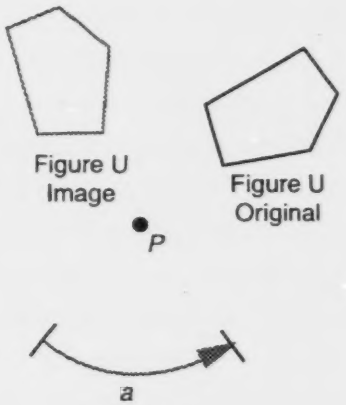


Figure U and its rotation image using arrow *a*.

Rotations help students understand angle measurement, symmetry, and slope.

x

Concept Development

STRAND and FOCUS	Lab A	PS 1	Lab B	PS 2	Lab C	PS 3	PS 4	Lab D	PS 5	PS 6	PS 7	PS 8	PS 9	PS 10
Introduction to Motion														
Moving Objects	1–5	2												
Orientation	6	2, 3, 4												
Congruence	1–4, 6													
Representation														
Representing Motion	5													
Reflection														
Properties of a Reflection		1	1											
Line of Reflection			1, 2	1, 2, 3	4	1, 4	1	4	1		2		2	
Reflection Image										1				
Comparing Motions												1		
Finding the Line of Reflection														1
Translation														
Properties of a Translation				4	1, 2	2								
Direction of Motion					3									
Distance of a Translation							4							
Translation Arrow							4	2	3	2, 3	1			
Translation Image												4	1	
Comparing Motions												1		
Multiple Translations													4	
Translation of a Translation														2
Rotation														
Properties of a Rotation						3	2, 3	1		4	4	2		
Center of Rotation								1						
Rotation Arrow									2, 4	4		3		
Magnitude of a Rotation								3			3		3	
Comparing Motions														
Arc Measure of a Rotation Arrow														3

Materials

Materials used throughout Motion Geometry

Geometric compass, 2 per group

MIRA®, 1 per student

Straight-edge*, 1 per student

Tracing paper**, 2–3 sheets per student per lesson

Special materials for Motion Geometry

Lab A

A variety of flat, traceable objects, 1 object per student

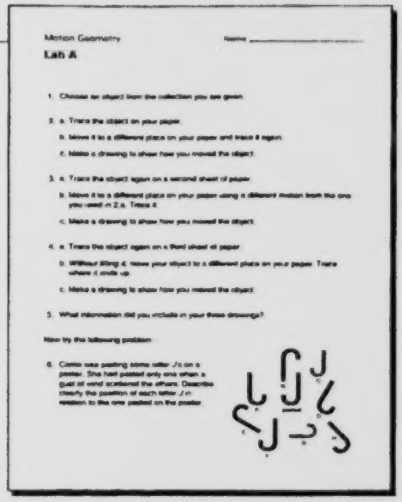
Blank half sheets of paper, 3 per person

*Straight-edges without unit markings are preferable to rulers. Blank index cards could also be used.

**Patty paper cut into 5-inch squares is an economical alternative to the art paper used for tracing.

Lab A

Recording and Describing Motions


Motion Geometry
Lab A

Lab A, Nos. 1–5

Strand: Introduction to Motion and Representation

Focus: Moving objects, congruence, and representation

Task: Moving objects and representing motion

PROBLEM

1. Choose an object from the collection you are given.
2. a. Trace the object on your paper.
   b. Move it to a different place on your paper and trace it again.
   c. Make a drawing to show how you moved the object.
3. a. Trace the object again on a second sheet of paper.
   b. Move it to a different place on your paper using a different motion from the one you used in 2.a. Trace it.
   c. Make a drawing to show how you moved the object.
4. a. Trace the object again on a third sheet of paper.
   b. Without lifting it, move your object to a different place on your paper. Trace where it ends up.
   c. Make a drawing to show how you moved the object.
5. What information did you include in your three drawings?

Teacher's Insight

This lab gives students an initial experience with motions and with representing them on paper. The first task leaves the motion entirely up to students. The second task asks them to use a different motion. The third task puts some constraint on what motion they can use. In the discussion students should begin to talk informally about the properties of the motions and the differences among the motions they used. They may begin to use some terms associated with motions, such as rotate, flip (reflection), slide (translation). Have them explain in their own words what these terms mean. They will come to understand them throughout this unit.

The directions intentionally do not specify what to draw to represent the motions, so students will use their own inventions. During the discussion they can explain what they were trying to convey in their representations and assess the effectiveness of different methods. Having such a discussion at this early point in the unit helps students see the rationale behind establishing conventions to represent motions.

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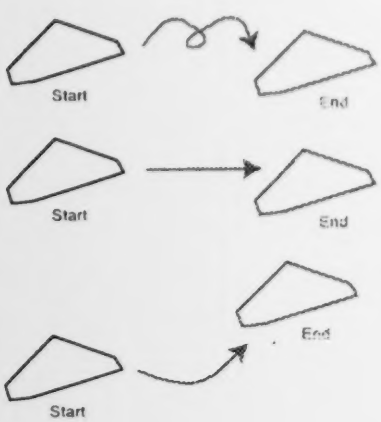
ANSWER

1–5. Arrows are often used to indicate the direction objects were moved. For example, if an object was slid downward, students may draw an arrow from the initial tracing and point it to the final tracing. If they raised the object off the paper, students may draw an arrow with a loop. Highlight the use of arrows that indicate the precise distance they moved an object. Such arrows resemble the arrows used in motion geometry. Also acknowledge any labels used to indicate the initial and final positions of the objects.

Student drawings may include



Students should also discuss what about the object changed and did not change when they moved it. Nothing about an object's features changes under a motion, but its orientation may change.

In the last problem students use their experience to interpret someone else's representation. This problem also invites them to use new terms that describe the orientation of a figure.

Discussion

Start the discussion by asking a few students to show their papers and describe the motions they used. Then have some students show their papers while others try to describe the motions used. Ask what kinds of information helped them to know what motion someone used and why it was helpful.

Have students look at someone's paper and try to recreate the motion used to move the object. Ask if some motions were easier than others to recreate. Why?

How did you know where the object started? How did you know where it ended up?

What makes one motion different from another motion?

What about the object changed? What did not change?

Lab A, No. 6

Strand: Introduction to Motion

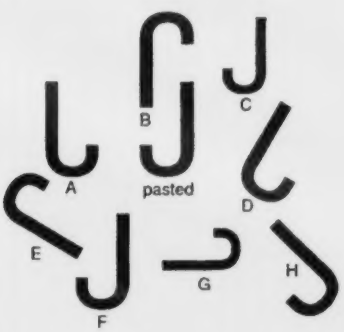
Focus: Orientation and congruence

Task: Comparing orientation with a given figure

PROBLEM

Now try the following problem:

6. Carrie was pasting some letter *J*s on a poster. She had pasted only one when a gust of wind scattered the others. Describe clearly the position of each letter *J* in relation to the one pasted on the poster.



Teacher's Insight

Students use a variety of ways to describe the orientation of each figure. They may describe the position of each figure in fractions or degrees of a turn. Some may use compass directions or describe the position in clock terms. This problem demonstrates that there can be different ways of describing something and highlights the importance of communicating clearly and checking to understand another person's viewpoint. Have students conjecture what motion resulted in the various orientations of the figures.

This problem also encourages students to consider what makes one figure the same as or different from another. All the *J*'s, except C and G, are congruent. Some students say that figures C and G moved backward and therefore appear smaller. Some students say these two figures were always smaller than the others. Students should distinguish between congruence and orientation.

ANSWER

6. The positions may be described as turned, flipped, or slid. Students may give specific amounts of turns in terms of degrees or fractions. For example, B was rotated a half-turn or 180° clockwise (or counter-clockwise). "Flipped to the left," may be used to describe the position of A. "Slid down to the left in a straight line," may describe F. Combinations of motions may also be used. For example, G was rotated 90° and flipped either left or right.

★ HOMEWORK

Problem Set 1

Discussion

Have students describe the position of each *J* compared to the one pasted on the poster.

How do you think the *J* moved in order to end up in that position?

Can you demonstrate how you think the *J* moved?

If students talk about some of the *J*'s (B, E, and H) being turned a certain fractional amount, ask how they decided how much.

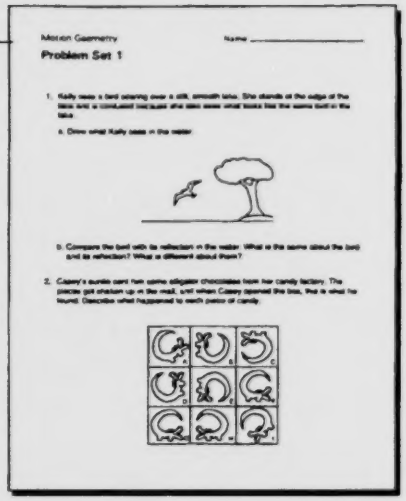
If students talk about figure B in terms of 180°, ask how they knew it turned 180°.

Are all the *J*'s the same? What makes them the same? What makes them different?

How did the gust of wind affect the figures?

Problem Set 1

Identifying Reflections



Problem Set 1, No. 1

Strand: Reflection

Focus: Properties of a reflection

Task: Comparing an original figure with its reflection image

PROBLEM

1. Kelly sees a bird soaring over a still, smooth lake. She stands at the edge of the lake and is confused because she also sees what looks like the same bird in the lake.

   a. Draw what Kelly sees in the water.

   b. Compare the bird with its reflection in the water. What is the same about the bird and its reflection? What is different about them?

Teacher's Insight

In this introductory problem students use their previous experience with reflections to think about the properties of a reflection. In reflections, the size and shape of a figure remain congruent, but the orientation changes. Everything on one side of the line of reflection is mirrored on the other, including the distance the objects are from the line of reflection. A common misconception is for students to think of shadows and not reflections. You can address this by asking them what the difference is between a shadow and a reflection or if they can tell what they are looking at. Then focus the discussion on the properties of a reflection.

Some students try to create the reflection of the bird with a freehand sketch. Some try to get a more accurate image by tracing the original bird. Then they fold their paper on the water line and retrace the image. Some rotate the tracing and get a 180° rotation instead of a reflection. One way to help students assess the accuracy of their method is to ask them to predict what the image should look like. Should it be the same size and shape? How should it face? Where should it be in the water? When students clarify what they are trying to accomplish, it is easier for them to assess their methods. When students share their solutions, it is always helpful to have them demonstrate how they did the reflection.

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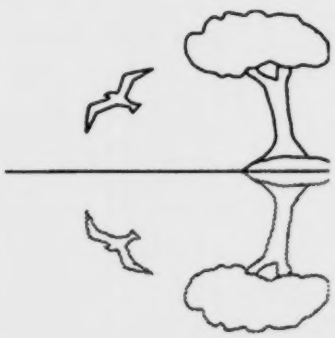
ANSWER

1. a.



b. The bird and its reflection are the same shape and size. The bird is real but the reflection is just an image; it looks real but it isn't. The reflection looks like an upside-down bird. The bird itself is flying with its right wing higher than its left, but the reflection makes the right wing look lower than the left.

Discussion

What would you expect the bird in the lake to look like?

How did you draw the reflected image?

What is different about the bird and its reflection?

What is the same about the bird and its reflection?

How did you decide where to put the image of the bird?

If students mention that they made the reflection drawing the same distance from the water line as the bird in the sky, ask why they think that should be so. How could you check the distance without using a ruler?

If students used the water line in their method, ask how the line helped them to create their reflected image.

If students used a folding method, ask where they folded the paper.

Have students agree on names for the actual figure (pre-image or original) and their reflection (image).

What are some ways to make the drawings more accurate?

Problem Set 1, No. 2

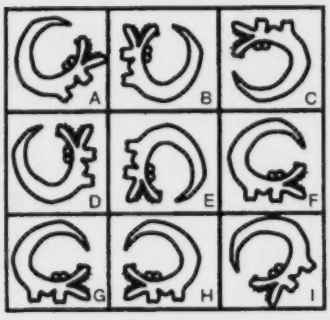
Strand: Introduction to Motion

Focus: Moving objects and orientation

Task: Describing motion

PROBLEM

2. Casey's auntie sent him some alligator chocolates from her candy factory. The pieces got shaken up in the mail, and when Casey opened the box, this is what he found. Describe what happened to each piece of candy.



Teacher's Insight

This problem highlights the need to make and agree on an assumption about the conditions of a problem before discussing its solution. In this problem no particular piece of candy is facing the "correct" way, but students must make an assumption before they can solve the problem. (Some students say that they did not know how the candy started out, so they could not solve the problem.) Students identify the assumptions they made to solve the problem. They listen to other viewpoints and they use a variety of ways to describe what happened to each piece of candy. This problem extends the experience of the *J* problem in the previous lab.

ANSWER

2. Multiple answers are possible. Most students begin by assuming alligator G is facing the original way and then describe the positions of the others by comparing each to alligator G. Descriptions include such words as turned, flipped over, upside-down, did not move, rotated 90°, rotated 180°.

Discussion

Have students explain their system for describing what happened to each piece of candy (e.g., in clock terms, fractions, or degrees) and check to see if others understand their descriptions of how the pieces are oriented.

How did you decide how the alligators were facing when the candy was mailed?

Did any candy arrive still facing the "right" way? If yes, what made you decide it was the "right" way?

How did you decide what happened to each piece of candy?

If students use fractions and degrees to tell how much the alligators rotated, be sure to probe understanding by asking what they meant by a $\frac{1}{4}$ turn. How do you know it rotated 90°?

What kind of motion(s) could have caused each piece to end up the way it did?

What is the same about all of the candies? What is different?

Problem Set 1, Nos. 3–4

Strand: Introduction to Motion

Focus: Orientation

Task: Comparing their bodies with reflection images of their bodies

PROBLEM

3. Look straight into the mirror.
   a. Draw your face.
   b. Close one eye. Draw your face again.
   c. Hold up your outspread hand with the palm facing the mirror. Draw your hand.

4. Write about your drawings.
   a. How are your drawings the same as your actual body?
   b. How are the drawings different from your actual body?
   c. What is special about a reflection?

ANSWER

3–4. Students should comment on how it seemed like the opposite side of their body was doing the action when they looked in the mirror. For example, when a student closes her left eye she will draw a picture of herself with her right eye closed. Students should also write about the properties of their reflections. For example, they may comment on how the reflection looked exactly like themselves, it was the size of their actual body, and their skin color and clothes were the same.

★HOMEWORK

None

Teacher's Insight

This drawing task gives students experience representing a three-dimensional object in a two-dimensional format. It highlights the properties of a reflection. Help them distinguish between the mathematical properties (orientation, size, etc.) and non-mathematical properties (how much the drawing resembles the person) and make comparisons between the real object and the drawing.

Discussion

Have students share their drawings and explain what they were trying to accomplish in making them.

What did you notice about the reflection?

How could you describe what happened in the mirror when you closed your eye? What happened when you held up your hand?

How does this problem remind you of the bird problem?

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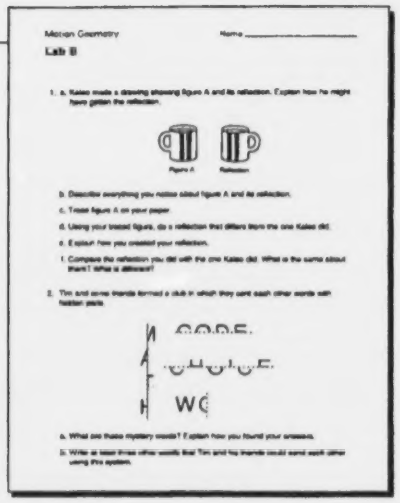
Lab B

Developing a Method for Reflections



Lab B, No. 1

Strand: Reflection

Focus: Properties of a reflection and line of reflection

Task: Creating methods to find a line of reflection and drawing the reflection images

PROBLEM

1. a. Kaleo made a drawing showing figure A and its reflection. Explain how he might have gotten the reflection.

   b. Describe everything you notice about figure A and its reflection.

Figure A Reflection

   c. Trace figure A on your paper.

   d. Using your traced figure, do a reflection that differs from the one Kaleo did.

   e. Explain how you created your reflection.

   f. Compare the reflection you did with the one Kaleo did. What is the same about them? What is different?

Teacher's Insight

The method for doing a reflection is related to the properties of a reflection. In this problem students explore properties of a reflection and methods for doing a reflection. All methods should use a line of reflection as the axis over which the original figure is reflected. The points along this axis remain fixed.

Students distinguish between different methods for doing a reflection and different reflections (outcomes). Different reflections (outcomes) result from using different lines of reflection. Different methods of doing a particular reflection should result in the same reflection image. Students can use a Mira®, reflecting tools that allow them to see the original figure and the reflected image at the same time. The Mira® is placed along the line of reflection. Students can also fold their paper along the line of reflection and, by tracing the original figure, create the image. Some students trace over the original figure with a soft pencil, fold the paper along the line of reflection, and use a rubbing technique to transfer the tracing of the original to the location for the image.

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ANSWER

Multiple solutions are possible, including the following:

1. a. Kaleo could have placed a reflecting tool like a Mira® on the right of figure A and then drawn the reflection where he saw it, to the right of the figure.

   b. The figure and reflection are the same size and shape. The figures seem to be facing in opposite directions.

   c. and d.

Figure A

   e. Hold a Mira® above the original figure and look at the reflection in the Mira®'s side. Trace the reflection onto the paper on the opposite side of the Mira®.

   f. The reflection here and the one Kaleo did are congruent to the figure. This reflection makes the mug upside-down and the one Kaleo did makes the mug face the opposite way. The Mira® needs to be held in a different place to get this reflection.

Discussion

Explain how Kaleo might have reflected figure A to get the image.

Could Kaleo have used another reflection method to get the same image? Why?

Explain how you did a reflection that differed from Kaleo's.

What makes your reflection different from his?

Was there anything the same about your reflection and Kaleo's?

What are some tools that help you get the most accurate reflection?

Compare the various methods for doing a reflection.

What method do you prefer? Why?

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Lab B, No. 2

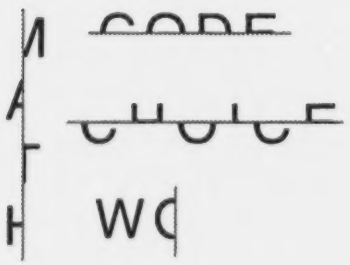
Strand: Reflection

Focus: Line of reflection

Task: Connecting the function of a mirror to that of a geometric line of reflection and understanding the properties of reflections

PROBLEM

2. Tim and some friends formed a club in which they sent each other words with hidden parts.



a. What are these mystery words? Explain how you found your answers.

b. Write at least three other words that Tim and his friends could send each other using this system.

ANSWER

2. a. MATH, CODE, CHOICE, WOW

b. Multiple answers are possible, including the following: MOMMY, TOT, HIDE

Teacher's Insight

Students enjoy working on this problem. For an additional challenge, have them write messages with this code or ask them to find the longest word for which this code will work.

Discussion

What is the mystery message system?

Did you notice anything different about the words? Did they all work the same way? How did you complete each word? How did you create new words to fit Tim's system?

How did you test your new words to be sure they fit the system?

How is Tim's system like the mug reflection in No. 1? How is it different?

Are these word reflections easier or harder to do? Why?

Will all words work this way? Why? Why not?

★HOMEWORK

Problem Set 2

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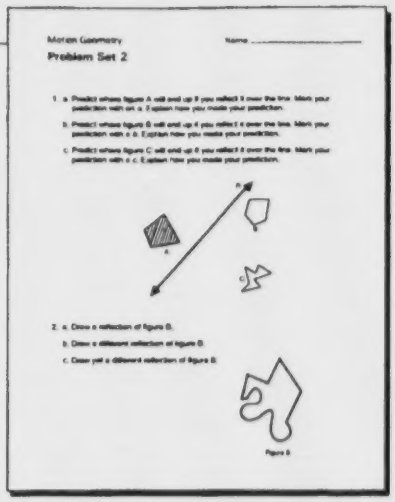
Problem Set 2

Reflecting



Problem Set 2, No. 1

Strand: Reflection

Focus: Line of reflection

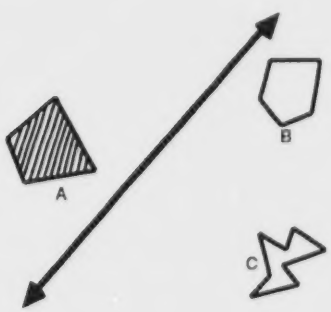
Task: Predicting how the line of reflection affects the location of a reflection image

PROBLEM

1. a. Predict where figure A will end up if you reflect it over the line. Mark your prediction with an *a*. Explain how you made your prediction.

   b. Predict where figure B will end up if you reflect it over the line. Mark your prediction with a *b*. Explain how you made your prediction.

   c. Predict where figure C will end up if you reflect it over the line. Mark your prediction with a *c*. Explain how you made your prediction.



Teacher's Insight

Predicting the outcome of a problem can help students devise a strategy for solving the problem and assessing their answer. Just as using number sense is valuable in judging the reasonableness of computation, using "motion sense" is useful in understanding a motion's properties and in assessing drawings of the motion. In this unit, as each of the rigid motions is introduced, there will be such a prediction problem. Students are expected to make their predictions without actually performing the motion. They should explain their thinking in making their predictions.

In this problem original figures lie on both sides of the line of reflection. The entire plane on one side of the line of reflection is reflected over the line to the other side and vice versa. The plane includes the figures and the area around them. In their predictions students should show they understand that in a reflection the distance between a figure and the line of reflection should be maintained and the image should be the same distance from the line of reflection as the original figure was.

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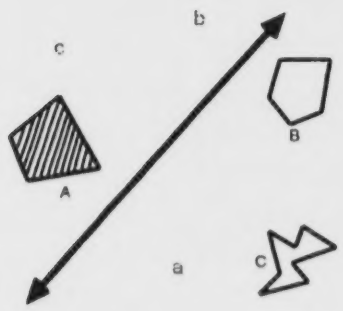
ANSWER

1. a–c.



Discussion

How did you make your prediction?

How did you decide where the images would be?

Why will some images be closer to (or farther from) the line than others?

If students did the reflection instead of predicting, ask if the image ended up where they expected it to.

Does this problem remind you of another problem? (Problem Set 1, No. 1) Explain how.

What is the purpose of the line of reflection?

Problem Set 2, No. 2

Strand: Reflection

Focus: Line of reflection

Task: Defining a *different reflection* and identifying the properties of a complete reflection drawing

PROBLEM

2. a. Draw a reflection of figure B.

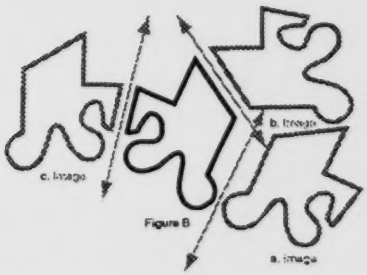
   b. Draw a different reflection of figure B.

   c. Draw yet a different reflection of figure B.

Figure B

ANSWER

2. a.–c. Multiple answers are possible, including the following:



Teacher's Insight

This problem gives students an opportunity to consider what makes one reflection differ from another and to review methods for doing the reflection. It is also a good problem for highlighting the features of a complete reflection drawing. Drawings should include the original figure (labeled), the line used to reflect the figure, and the reflection image (labeled). Students sometimes omit the line of reflection or the original figure in their drawings. Emphasize that a person can assess the correctness of a drawing only if all three elements appear in the solution. Students should show understanding that the size and shape of a reflection image are the same as those of the original and that a different reflection requires using a different reflection line and produces a reflection image in a different place.

Discussion

Have students explain how they did their reflections.

What did you do to get a different reflection?

How did you decide where to draw your line of reflection?

How did you use the line of reflection to reflect the original?

What is the same about all of your reflected images? What is different about them?

What do you need to do a reflection?

How many *different* reflections of figure B are possible?

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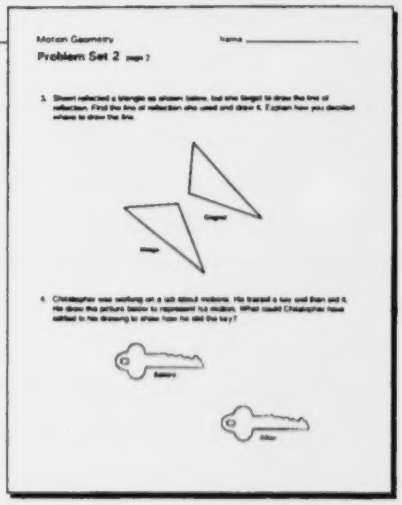


Problem Set 2, No. 3

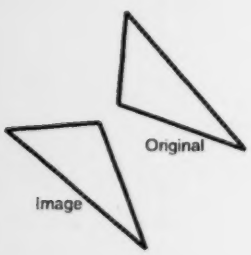
Strand: Reflection

Focus: Line of reflection

Task: Finding the line of reflection when given the original figure and its image

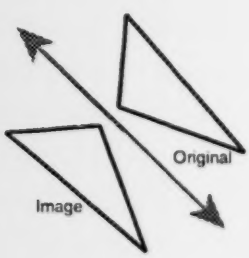
PROBLEM

3. Sherri reflected a triangle as shown below, but she forgot to draw the line of reflection. Find the line of reflection she used and draw it. Explain how you decided where to draw the line.



ANSWER

3. The line of reflection is where the paper must be folded so that the original and the image map onto each other. It is located in the middle of the original and its image. Although there are several methods to finding the line, this answer is unique.



Teacher's Insight

In this problem students use reversibility to find the line of reflection. Most students fold the paper so that the original and the image coincide. The discussion should address the idea that the line of reflection is unique. That is, only one line of reflection will result in that image. You can also draw students' attention to the distance between the line of reflection and the original and image figures.

Discussion

How did you decide where to draw the line of reflection?

Can you find a different line that works? Why or why not?

What do you notice about the location of the line of reflection?

Problem Set 2, No. 4

Strand: Translation

Focus: Properties of a translation

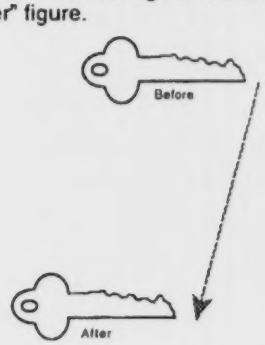
Task: Indicating and explaining the motion of a slide in a drawing

PROBLEM

4. Christopher was working on a lab about motions. He traced a key and then slid it. He drew a picture to represent his motion. What could Christopher have added to his drawing to show how he slid the key?

ANSWER

4. Multiple answers are possible but should include an arrow pointing from the "Before" figure to the "After" figure.



★HOMEWORK

None

Teacher's Insight

This is the first problem about slides, called translations. In a translation the figure moves a specified distance in a straight path. The image is congruent to the original figure. The first translation problems use the term *slide* because it is more familiar to students. Students draw on their experience from Lab A to add markings that represent a slide. In discussing what they included to show the motion, students can focus on the properties of a slide. Usually they use arrows to indicate the direction of the motion, but their arrows do not represent the length of the slide.

A translation arrow will show the direction and the distance of a straight slide. This idea will be developed in future problems.

Discussion

Have students share what they added to the drawing and explain how their markings represent the motion.

How would you describe the slide?

What do you notice about the original figure and the image after it was slid? What stayed the same? What changed?

How far did the key slide?

How does the arrow resemble the slide?

Could your arrow indicate a location for a different "After" key?

Is there an arrow that just indicates this "Before" key to this "After" key?

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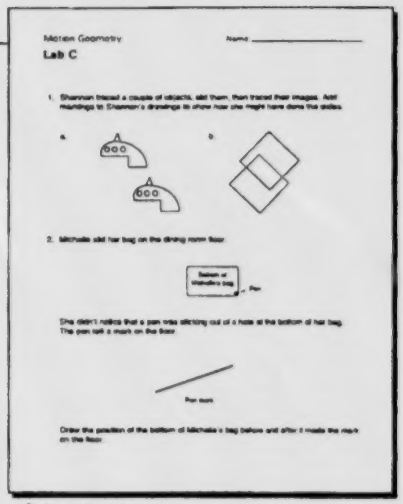
Lab C

Identifying Translations



Lab C, No. 1

Strand: Translation

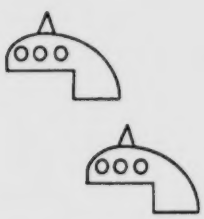
Focus: Properties of a translation

Task: Assuming an original and its image and using notation to show the direction and distance an object was slid

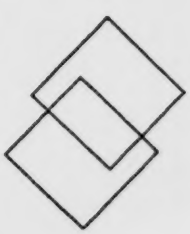
PROBLEM

1. Shannon traced a couple of objects, slid them, then traced their images. Add markings to Shannon's drawings that show how she might have done the slides.

a.



b.



Teacher's Insight

This problem deliberately omits the labels for the original and image. This omission highlights the importance of labeling in motion drawings. Students typically use arrows to show the direction of the slide but not its length. Future problems will develop this concept. To help students begin to think about the length of a slide, you can highlight the different distances of the two slides and ask students how they might show the differences. Labeling is important because it shows where the motion started.

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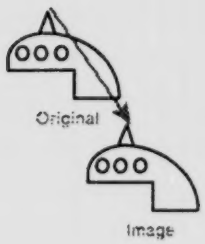
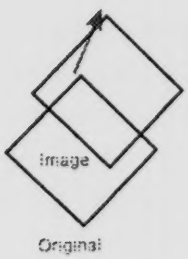
ANSWER

1. a. Answers to highlight are ones that identify one figure as the original and the other figure as the image, that include an arrow pointing from the original to the image, and that approximate the distance between corresponding points.



b.



Discussion

In each drawing, how did you decide which was the original and which was the image?

What did you include in the drawing to identify which figure you considered the original and which figure you considered the image?

How could someone looking at your paper know how Shannon did the slides?

Did both figures slide the same distance? How do you know? How could someone else know?

If students add arrows to drawings to show path, call on other students to re-create the slide.

If students talk about the properties of the figures (same size, same orientation), ask how they can tell the object was not reflected.

What is a slide?

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Lab C, No. 2

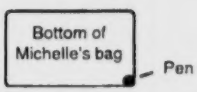
Strand: Translation

Focus: Properties of a translation

Task: Distinguishing the motion of a translation from other kinds of slides

PROBLEM

2. Michelle slid her bag on the dining room floor.

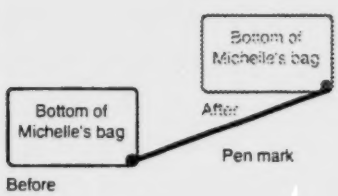


She didn't notice that a pen was sticking out of a hole at the bottom of her bag. The pen left a mark on the floor.

Draw the position of the bottom of Michelle's bag before and after it made the mark on the floor.

ANSWER

2. Look for drawings that place the holes of the two bags at opposite ends of the pen mark. The before and after positions should be nearly parallel, indicating a straight slide.



Teacher's Insight

This problem is key to helping students define a translation, discuss the properties of the motion, and distinguish it from other sliding motions they may have experienced, such as a twisting slide. The problem does not specify the direction that Michelle slid her bag. That is, it does not give complete information about the path of the slide or indicate the bag's position before or after the slide. Most students draw the before and after pictures of the bag using the position as given in the problem. Because the pen mark is straight, they assume the bag stayed in the same position throughout the motion. If students do not raise other possibilities, such as that the bag spun around the pen as it slid or it ended up in a turned position when it stopped, you should. Discussing such alternatives helps students define a translation and distinguish it from other sliding motions.

Discussion

Have students share and discuss a variety of answers.

What does the pen mark tell you?

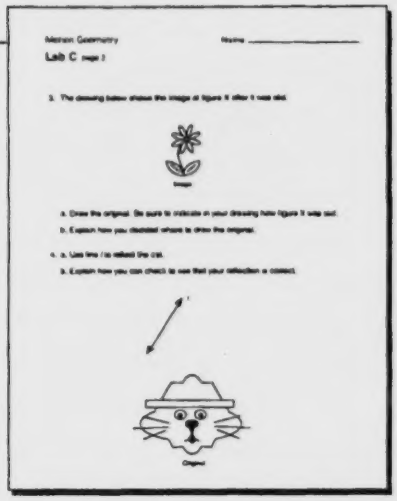
How did the bag move?

What do you notice about the bottom of the bag?

Could the bag end up in a different position?

How could you describe the motion used for the different possibilities presented?

What makes a translation different from a slide?



Lab C, No. 3

Strand: Translation

Focus: Direction of motion

Task: Using reversibility to decide the location of an original figure and getting an informal idea of a translation arrow

PROBLEM

3. The drawing below shows the image of figure X after it was slid.

Image

a. Draw the original. Be sure to indicate in your drawing how figure X was slid.

b. Explain how you decided where to draw the original.

Teacher's Insight

This problem presents the image of a slide. Students must decide where to draw the original figure and how to draw the motion. When students share their solutions, have them explain how they knew where to draw the original figure. They usually say that since the problem did not say where to draw the original, they could draw it anywhere. Some students say they could not do the problem. Many students base their decisions on where the original fits on their paper. Use this discussion to highlight the importance of including some indication of how far the figure slid.

We use a picture of a familiar object to help students talk about the difference between the way objects behave in the real world and how they behave in the mathematical world of translations. Students may need help in understanding the difference between how a solid object moves and a two-dimensional representation of the motion. Talk about what happens when a real flower moves and what is represented as a translation on paper. For instance, some students may think the stem should bend as it is being slid because a real flower's stem bends with the wind.

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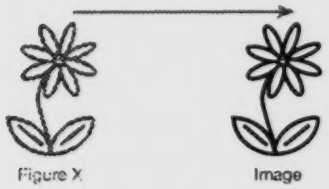
ANSWER

3. a. The original position of the flower could be anywhere. It should be labeled figure X or original. An arrow should be used to indicate the direction and possibly the distance the figure was slid.



b. Students generally state that the flower could have slid from anywhere because no arrow was drawn.

DISCUSSION

Have students share and discuss a variety of answers.

How does your drawing show what happened to the figure when it slid?

How did you show how the figure was slid?

How did you decide where figure X started from?

How did you make a drawing of its position? Did you trace anything?

How did you know which way the flower is supposed to slide?

What stayed the same?

What changed?

Why is the figure labeled "Image"? Are there other possible original positions? Are there any positions that aren't possible?

Lab C, No. 4

Strand: Reflection

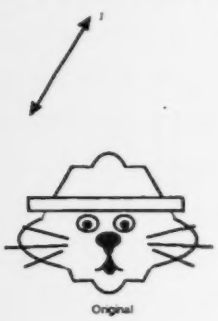
Focus: Line of reflection

Task: Reflecting a figure using a given line of reflection

PROBLEM

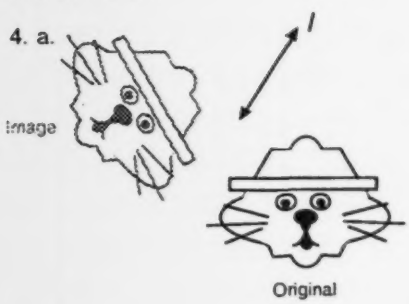
4. a. Use line *l* to reflect the cat.

   b. Explain how you can check that your reflection is correct.



ANSWER

4. a.



b. You can check the reflection by folding the paper on the line of reflection and seeing if the original and the image overlap each other perfectly.

★HOMEWORK

Problem Set 3

Teacher's Insight

The oblique position of the cat in relation to the line of reflection puzzles some students. Some students move the cat or the line so they are aligned more squarely. If students move either, help them see how doing so changes the conditions of a problem. Students may not know that they can extend the line to make reflecting the cat easier. The discussion should highlight the idea that a line extends infinitely in both directions and clarify how its representation fits the definition.

Some students will sketch the image of the cat where they think it should fall instead of actually reflecting it and drawing the reflection. Encourage them to take the time to do their reflections as accurately as possible by using one of the methods discussed earlier.

Discussion

Have students demonstrate the method they used to reflect the cat.

How did you do your reflection?

How did you use line *l* to do the reflection?

Is there anything we can change about what is given in this problem? Can the line be moved to a different place? Can the line be extended? Why?

Did the length of the line matter? How did it affect the reflection?

Is your answer unique? Why should we all get the same image?

Why might some people think this problem is "tricky"?

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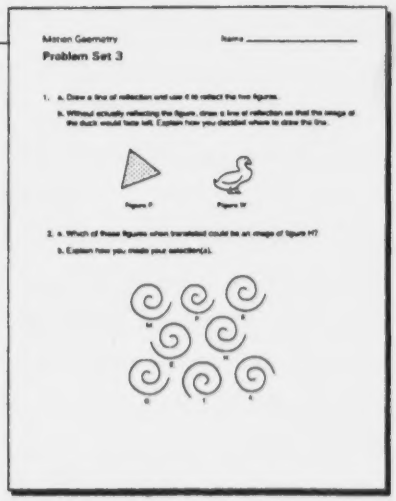
Problem Set 3

Developing a Method for Translations



Problem Set 3, No. 1

Strand: Reflection

Focus: Line of reflection

Task: Selecting lines of reflection and judging the effects of the placement of the lines on the images

PROBLEM

1. a. Draw a line of reflection and use it to reflect the two figures.

   b. Without actually reflecting the figure, draw a line of reflection so that the image of the duck would face left. Explain how you decided where to draw the line.

Figure P

Figure W

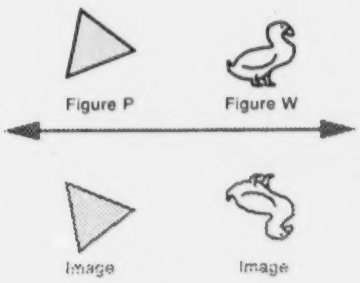
Teacher's Insight

In the first part of the problem students can draw the line of reflection wherever they wish. Some may reflect each figure separately. Point out that the problem gives them the two figures to reflect; reflecting them separately changes the given problem. To help students understand, you could point out that reflecting the figures separately is like changing the problem 25 + 17 to 2 + 1 and 5 + 7.

In 1.b. students use their motion sense and their knowledge about the properties of reflections to predict a reflection that will give the desired results.

ANSWER

1. a. Multiple answers are possible, including the following:



b. Multiple answers are possible, including the following:

Discussion

How did you decide where to draw your line of reflection in 1.a.? Is your answer unique? How many different ways could you solve 1.a.?

How did you decide where to draw the line of reflection in 1.b.? Is your answer unique? How many different ways could you solve 1.b.?

How does the line help you reflect the figures?

Explain how the location of the line of reflection affects the image.

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Problem Set 3, No. 2

Strand: Translation

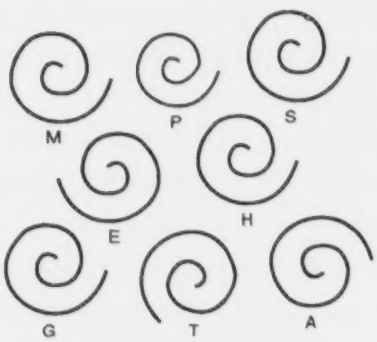
Focus: Properties of a translation

Task: Identifying and describing properties of translation images

PROBLEM

2. a. Which of the figures when translated could be an image of figure H?

   b. Explain how you made your selection(s).



ANSWER

2. a. S, M, and G.

   b. These images look like straight slides because they are facing the same way as figure H. This could be checked by tracing figure H and sliding it in a straight line to see if it will match up with the others.

Teacher's Insight

By contrasting examples and non-examples of translation images, students strengthen their understanding of the motion's properties. In the discussion ask students to explain the reasoning they used to choose the examples of translation images and the reasons the other figures are not examples. Also address the methods students used to test their perceptions. Some students will base their decisions only on observation. This is a good beginning, but they should also know how to use tests that are more mathematically certain. Help them identify a more robust strategy for testing their first guesses. One such test would be to trace a figure and translate it to see if it maps on to figure H. If students talk about the possibility that figure P is an image of a translation moving backward, point out that the paper is a two-dimensional context. The motion they are describing calls for a three-dimensional context.

Discussion

How did you decide which were translation images of figure H?

What did you notice about the figures you chose as translation images of figure H?

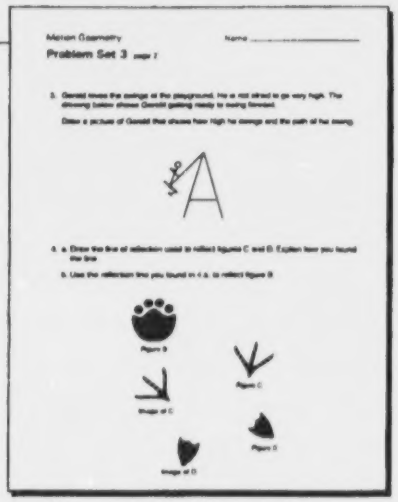
How did you test the images to see if they really were translation images?

How did you decide that the other figures were not translation images of figure H?

Why isn't figure T a translation image of figure H? What about figure P?

What is an image?

40



Problem Set 3, No. 3

Strand: Rotation

Focus: Properties of a rotation

Task: Identifying the general motion of a rotation around a center

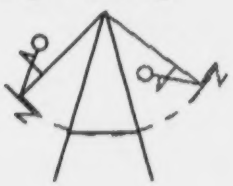
PROBLEM

3. Gerald loves the swings at the playground. He's not afraid to go very high. The drawing below shows Gerald getting ready to swing forward.

   Draw a picture of Gerald that shows how high he swings and the path of his swing.

ANSWER

3. Multiple answers are possible, including the following:



Teacher's Insight

This problem introduces the rotation strand by focusing on the properties of a rotation, such as the shape of the path. Students discuss details of the properties in subsequent problems. In a rotation the original figure moves in a circular path around a fixed center of rotation. A rotation image is congruent to the original figure.

Students first encounter a rotation in the context of a swing. The swing moves in an arc whose center of rotation is where the swing's chains attach to the frame. The problem's open-ended nature elicits students' prior experience and lets you assess what they already know about rotations. Students usually use a curved arrow to represent the path of the rotation.

Discussion

How did you *decide* how high Gerald swung?

How did you *show* how high he swung?

In what direction did he swing? How could you show it?

How might you describe this movement?

What shape did Gerald's path resemble?

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Problem Set 3, No. 4

Strand: Reflection

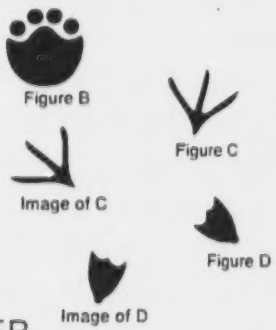
Focus: Line of reflection

Task: Identifying the line of reflection and using it to create a reflection image

PROBLEM

4. a. Draw the line of reflection used to reflect figures C and D. Explain how you found the line.

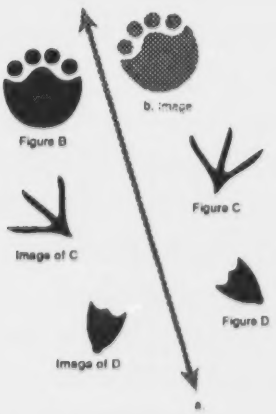
   b. Use the reflection line you found in 4.a. to reflect figure B.



ANSWER

4. a. The line of reflection is found by folding the paper so that the figures map over their corresponding images.

   b.



★HOMEWORK

Problem Set 4

Teacher's Insight

Students use reversibility to find the line of reflection and then use the same line to reflect a given figure. In this drawing two original figures are on one side of the line of reflection and the third original figure is on the left. Help students focus on the details of the given drawing, especially the labeling. They should remember to label the drawing that represents their solution carefully.

Discussion

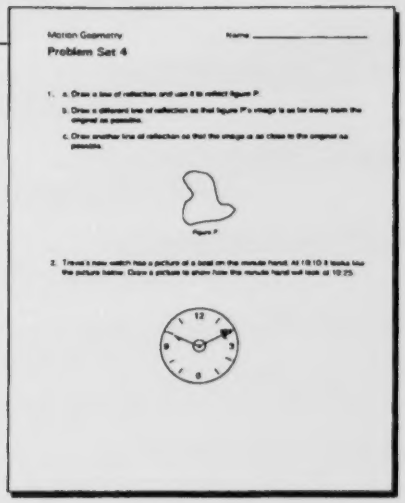
How did you know where to draw the line of reflection?

Is your answer unique?

Make as many observations as you can about the original and the image of a reflection.

Problem Set 4

Identifying Rotations


Motion Geometry Name _____

Problem Set 4

1. a. Draw a line of reflection and use it to reflect figure P.

b. Draw a different line of reflection so that figure P's image is as far away from the original as possible.

c. Draw another line of reflection so that the image is as close to the original as possible.

Figure P

2. There's a new watch that has a picture of a bee on the minute hand. At 10:10 it looks like the picture below. Draw a picture to show how the minute hand will look at 10:25.

Problem Set 4, No. 1

Strand: Reflection

Focus: Line of reflection

Task: Placing a line of reflection to provide a desired result and comparing the distances between the line of reflection, the original, and the image

PROBLEM

1. a. Draw a line of reflection and use it to reflect figure P.

   b. Draw a different line of reflection so that figure P's image is as far away from the original as possible.

   c. Draw another line of reflection so that the image is as close to the original as possible.

Figure P

Teacher's Insight

In this problem students explore how the placement of the line of reflection relative to the original figure affects the image. Many students believe that the farthest possible placement is the limit of their paper and that the closest placement is just touching the figure. But the line can be placed anywhere, even through the figure itself. If no one raises a question about such limitations, ask if there have to be limits.

Note: When the line of reflection passes through the figure, students must be sure to complete the reflection of the figure from both sides of the line. In Lab D, No. 4, students will see such a problem.

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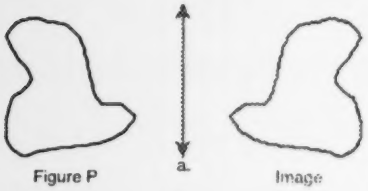
ANSWER

1. a. Multiple answers are possible, including the following:



   b. The farther the line is from the original, the farther away the image will be. Look for lines that are drawn close to edges of the paper.

   c. The closer the line is to the original, the closer the image will be. Look for lines that are tangent to or intersecting the figure.

Discussion

Have students share and discuss a variety of answers.

How did you decide where to draw the lines of reflection?

What is the farthest from the original you could draw a line of reflection?

What is the closest you could draw a line of reflection?

How does the placement of the line of reflection affect the image?

Problem Set 4, No. 2

Strand: Rotation

Focus: Properties of a rotation

Task: Drawing the image of a rotation, discussing the characteristics, describing the path, and identifying the center

PROBLEM

2. Travis's new watch has a picture of a boat on the minute hand. At 10:10 it looks like the picture below. Draw a picture to show how the minute hand will look at 10:25.

ANSWER

2.

Teacher's Insight

In this problem the boat rotates about the center of the clock. Its position at 10:10 is the original; its position at 10:25 is the rotation image. The problem gives students the opportunity to discuss the properties of the motion (it is a circular path), to learn how to do a rotation, and to observe the relationship of the image to the original figure.

Discussion

How did you know where to draw the minute hand?

How did the boat move?

How would you describe the path?

What changed or moved?

What did not move?

In what ways is a rotation like a clock?

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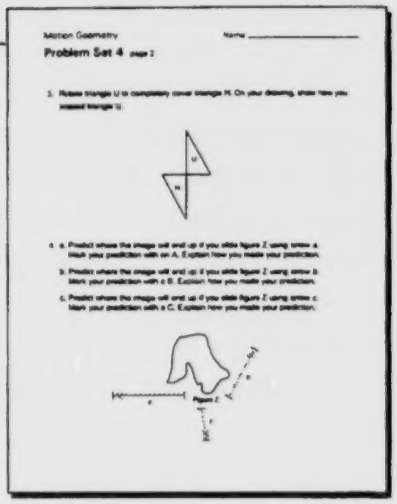

Problem Set 4, No. 3

Strand: Rotation

Focus: Properties of a rotation

Task: Comparing the effects of rotating in a different direction with the same center

PROBLEM

3. Rotate triangle U to completely cover triangle H. On your drawing, show how you rotated triangle U.

ANSWER

3. Rotate triangle U 180° around the point where the two triangles connect. The arrow could go either clockwise or counterclockwise.

Teacher's Insight

Students can rotate triangle U 180° clockwise or counterclockwise to achieve the results. Have them show their rotations and explain how their drawings represent the way they did the rotation. In discussing how much they rotated triangle U, students often use fractions or degrees. Whichever way they talk about the amount of rotation, be sure to ask them to explain how they knew how much the rotation was. When discussing what changed and what did not change, highlight the fact that the center remains fixed.

Discussion

How did you decide to rotate triangle U?

How much did you rotate triangle U?

Is your answer unique?

What is the difference between rotating the triangle clockwise or rotating it counterclockwise?

Why could you have rotated triangle U either clockwise or counterclockwise to cover triangle H?

What changed when you rotated triangle U? What did not change?

What moved when you rotated triangle U? What did not move?

Can any rotation be performed in two different directions? Why? Why not?

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Problem Set 4, No. 4

Strand: Translation

Focus: Distance of a translation and translation arrow

Task: Deciding on the function of a translation arrow and predicting the results of translating a figure using given translation arrows

PROBLEM

4. a. Predict where the image will end up if you slide figure Z using arrow *a*.

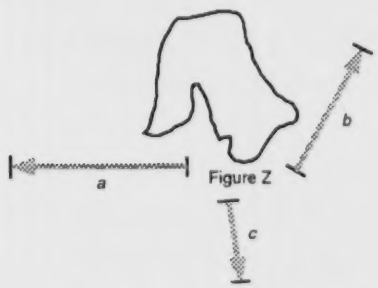
   Mark your prediction with an A. Explain how you made your prediction.

   b. Predict where the image will end up if you slide figure Z using arrow *b*.

   Mark your prediction with a B. Explain how you made your prediction.

   c. Predict where the image will end up if you slide figure Z using arrow *c*.

   Mark your prediction with a C. Explain how you made your prediction.

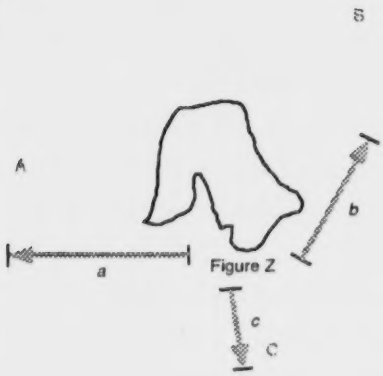


Teacher's Insight

The key point in this problem is how the translation arrow functions in a translation. The translation arrow shows the direction and the distance the original figure moves. The problem encourages students to use their motion sense to predict the outcomes of using various translation arrows and to focus on the details of the arrow. Students should not actually do the translations for homework, but in the class discussion you can have them test their predictions by performing the translations.

ANSWER

4.



★HOMEWORK

None

Discussion

What did the arrows tell you?

Demonstrate how you used them.

How did you predict where the images would end up?

How did you know in which direction the figure would move?

How did you know how far the figure would move?

After students complete the translations in class, ask them how accurate their predictions were.

Have students demonstrate their methods for doing the translation. Then ask how far the figure moved in each translation. (The length of the arrow)

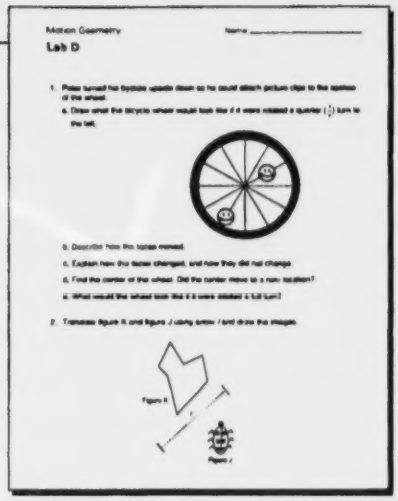
How did you know when to stop moving the figure?

Could the figure end up anywhere? Why not?

What does the arrow tell you about the path of a translation?

Lab D

Developing a Method for Rotations



Lab D, No. 1

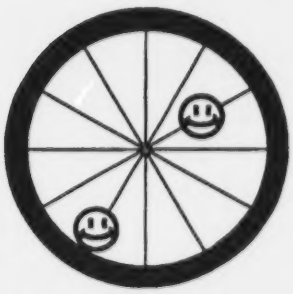
Strand: Rotation

Focus: Properties of a rotation and center of rotation

Task: Relating the movement of a familiar object to the geometric motion of rotation and relating the center of the wheel to the center of rotation

PROBLEM

1. Peter turned his bicycle upside down so he could attach picture clips to the spokes of the wheel.
   a. Draw what the bicycle wheel would look like if it were rotated a quarter ($\frac{1}{4}$) turn to the left.



   b. Describe how the faces moved.
   c. Explain how the faces changed, and how they did not change.
   d. Find the center of the wheel. Did the center move to a new location?
   e. What would the wheel look like if it were rotated a full turn?

Teacher's Insight

In this problem students investigate the properties of rotation and use terms to describe a particular rotation. They need to clarify whether "turn left" means the top or the bottom of the wheel moves left. The image will depend on what they decide. Later problems will demonstrate the use or function of a rotation arrow to identify a starting point.

Students may think the faces "change" if they picture them sliding along the spoke. Have them focus on the size, shape, and features of the faces when deciding whether they change. The center does not move or change in any way.

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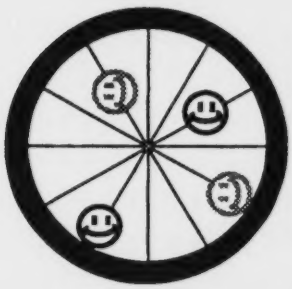
ANSWER

1. a. If "rotate to the left" means the wheel rolls to the left, then these are the answers:



b. The faces are turned on their side after the quarter turn.

c. The location of the face changed. Each one looks as if it was moved three wires over. The faces are on the same wire of the wheel, and they are the same shape and size.

d. No, the center holds the wheel to the bike so it stays stationary.

e. The wheel would look the same as it did in its starting position.

Discussion

How did you decide how much a quarter turn was?

How did you decide in what direction to turn the wheel?

How did your decision about which way to turn the wheel affect the image?

How did you find the center of the wheel?

How did you decide what a full turn was?

How can we change the problem so there is no confusion about the direction to turn the wheel?

Lab D, No. 2

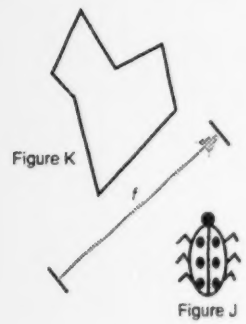
Strand: Translation

Focus: Translation arrow

Task: Demonstrating the function of a translation arrow

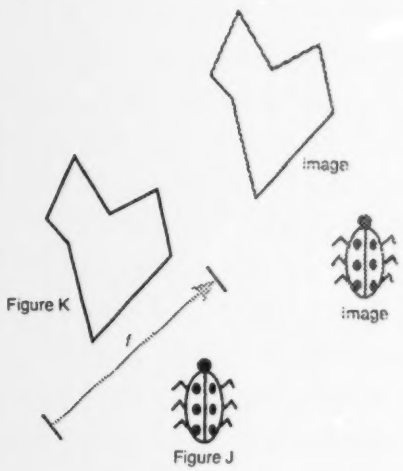
PROBLEM

2. Translate figure K and figure J using arrow *f* and draw the images.



ANSWER

2.



Teacher's Insight

This problem gives students experience with translating a figure using a specified translation arrow. Have them demonstrate their methods for doing the translation. Review the information the arrow gives them about the motion. Students sometimes translate each figure separately. If someone translates both figures at the same time, ask whether the result would be the same and how efficient translating both figures in the same motion might be. If no one translates both figures at the same time, ask students if it is possible and then have them do it to see that the outcome is the same.

Discussion

Have students demonstrate the motion.

If needed, have students practice doing the translation.

How did you use the arrow?

How did you know where to start translating?

How did you know when to stop translating the figures?

How far did the bug move?

How far did figure K move?

What information do you get from the translation arrow?

How does that information help you do the translation?

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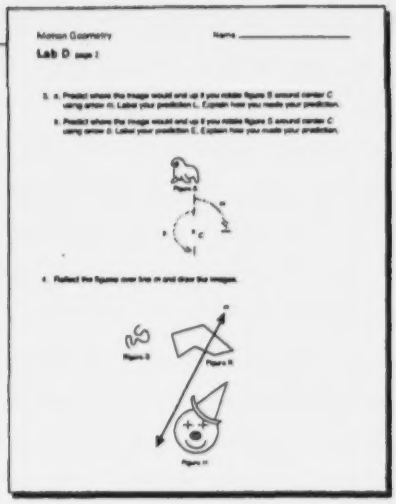


Lab D, No. 3

Strand: Rotation

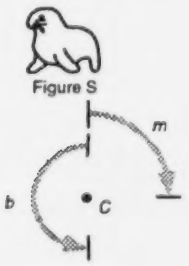
Focus: Magnitude of a rotation

Task: Predicting the location of a rotation image

PROBLEM

3. a. Predict where the image will end up if you rotate figure S around center *C* using arrow *m*. Label your prediction L. Explain how you made your prediction.

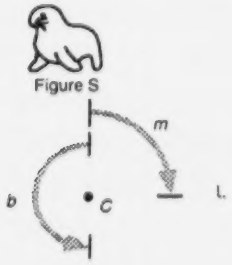
   b. Predict where the image will end up if you rotate figure S around center *C* using arrow *b*. Label your prediction E. Explain how you made your prediction.



ANSWER

3. a. The arrow is a quarter turn, so figure S should rotate that much. If the starting position is thought of as 12 o'clock, then a quarter turn is 3 o'clock.

   b. The arrow is a half turn going counterclockwise, so figure S should rotate from the 12 o'clock back to the 6 o'clock position.



Teacher's Insight

Students use their motion sense to predict the outcomes of rotations. Arrows *m* and *b* represent a quarter turn and a half turn, respectively. (Most students are familiar with these portions of a circle.) They may describe the turn in fractions or degrees. Whichever way they choose to express the amount of rotation, ask how they knew that was the amount. Students can test their predictions by performing the rotation during the discussion.

Discussion

How did you make your predictions?

How did you decide how far the figures would rotate?

Did you use the arrows? If so, how?

If students used fractions to make their predictions, ask how they knew arrow *m* was a quarter turn and arrow *b* was a half turn.

How did using the fractions help you?

How could you test your predictions?

Lab D, No. 4

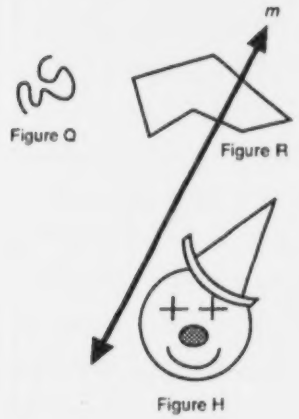
Strand: Reflection

Focus: Line of reflection

Task: Reflecting figures on both sides of the line of reflection, and a figure that is intersected by the line of reflection

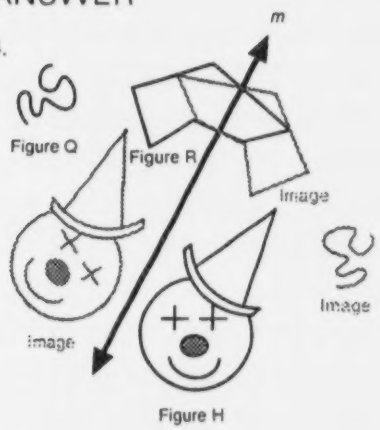
PROBLEM

4. Reflect the figures over line *m* and draw the images.



ANSWER

4.



★HOMEWORK

Problem Set 5

Teacher's Insight

Students may have difficulty reflecting the figure lying directly on the line of reflection. Check to see that they complete the reflection on both sides of the line.

Discussion

Have students demonstrate and describe their methods for reflecting the figures.

Is anything different about the drawing in this problem?

Did you use the same method to reflect all the figures?

What do you notice about the location of each figure's image in relation to the line of reflection?

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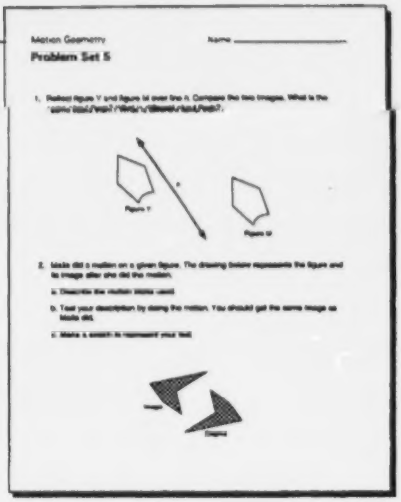
Problem Set 5

Using Motion Notation



Problem Set 5, No. 1

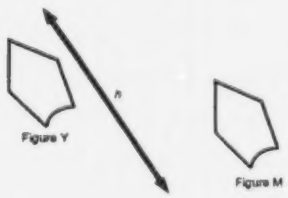
Strand: Reflection

Focus: Line of reflection

Task: Comparing the reflection of figures located at different distances from the line of reflection

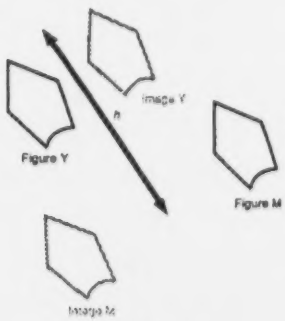
PROBLEM

1. Reflect figure Y and figure M over line *h*. Compare the two images. What is the same about them? What is different about them?



ANSWER

1. Figures Y and M are congruent and their images are also congruent. Figure Y is close to the line of reflection so its image is just as close to the line of reflection. Figure M is farther from the line of reflection. Its image is just as far from the line of reflection.



Teacher's Insight

This problem gives students the opportunity to review the idea that the entire plane is reflected when the figure is reflected. Most noticeably, the distance between the figure and the line of reflection is reflected along with the figure. Some students may say that the resulting drawing looks like a drawing of a translation. Ask what they noticed to make them say that, and have them focus on what "tool" included in the drawing tells them that the drawing represents a reflection (the line).

Discussion

What is the same about the reflections of figures Y and M?

What is different about the reflections?

Why is the image of one figure farther from the line than the other?

What other motion does this look like? What features tell you that it isn't a translation?

Problem Set 5, No. 2

Strand: Rotation

Focus: Rotation arrow

Task: Hypothesizing about a motion and testing the hypothesis

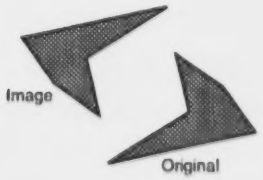
PROBLEM

2. Malia did a motion on a given figure. The drawing below represents the figure and its image after she did the motion.

   a. Describe the motion Malia used.

   b. Test your description by doing the motion. You should get the same image as Malia did.

   c. Make a sketch to represent your test.

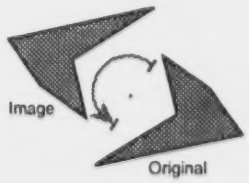


ANSWER

2. a. A rotation.

   b. Estimate the center of rotation by finding a point halfway between the original and image. Mark the center and draw a 180° rotation arrow around it. Then trace the original, center, and arrow, and rotate the tracing along the arrow while holding the center in place.

   c.



Teacher's Insight

In this problem students use reversibility to analyze a drawing and determine the motion it represents. The drawing can represent only a rotation (180°). With any other motion, one would have to do a combination of motions to achieve the same image. The discussion should highlight the difference between a single motion and a combination of motions, even if the results are the same. Students may have difficulty approximating the center of rotation used in this motion. Have them share the methods they used to test the motion they thought the drawing represented.

Discussion

How did you decide what motion Malia used?

What did you do to test your description?

How does your drawing represent your test?

What are you sure about the motion Malia did?

What aren't you sure about it?

How did you decide that Malia didn't use certain other motions?

How does testing your first guess help you know whether you were correct?

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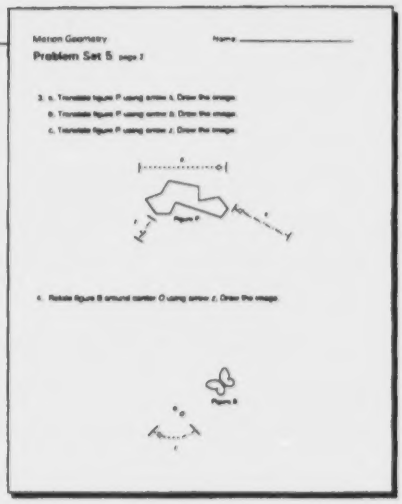


Problem Set 5, No. 3

Strand: Translation

Focus: Translation arrow

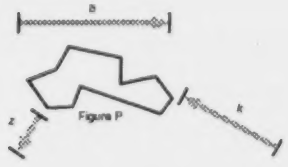
Task: Translating a figure using arrows of various lengths

PROBLEM

3. a. Translate figure P using arrow *k*. Draw the image.

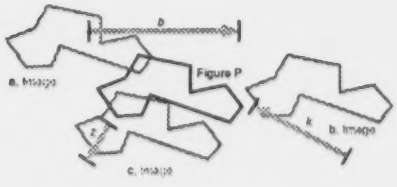
   b. Translate figure P using arrow *b*. Draw the image.

   c. Translate figure P using arrow *z*. Draw the image.



ANSWER

3. a.-c.



Teacher's Insight

Students need to clarify the mechanics of doing a translation: using the arrows, placing arrows on the paper, and seeing how placement affects the location of the image. Help students check that they are correctly labeling the original and the image by checking the direction of the arrow.

Students may have difficulty with arrow *k* because it is "behind" the original figure. They may have difficulty with arrow *z* because it is not long enough for the image to completely clear the original.

Discussion

Have students demonstrate and discuss the methods they used to translate the figure.

How did the arrows affect the translation?

How "far" did the figure move using arrow *b*?

How "far" did the figure move using arrow *k*?

How "far" did the figure move using arrow *z*?

Do you have to use a translation arrow where it is drawn on the paper? Why?

If students talk about overlap between the original and the image (using arrow *z*), ask why the figures overlap. Is it okay that the original and the image overlap?

Problem Set 5

What determines the location of the image?

What is it about these arrows that might tell us that they describe the distance of the translation? (The arrows are of different lengths.)

Problem Set 5, No. 4

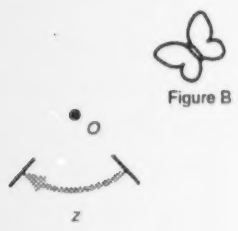
Strand: Rotation

Focus: Rotation arrow

Task: Rotating a figure with a specified center and rotation arrow

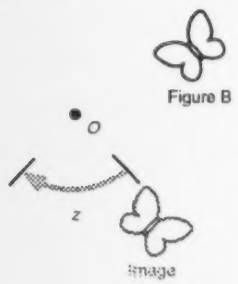
PROBLEM

4. Rotate figure B around center *O* using arrow *z*. Draw the image.



ANSWER

4.



★HOMEWORK

Problem Set 6

Teacher's Insight

Students will have to spend some time working on the mechanics of doing the rotation: tracing the arrow, the center, and the figure on paper, using the arrow, and discovering how the arrow affects the location of the image. This problem focuses students on the purpose of the center of rotation. The center is fixed. It "anchors" the motion so that the rotation moves in a true arc. Without a center there could be "wobble" in the motion, and it would not be a rotation.

Discussion

Have students demonstrate and discuss the methods they used to rotate the figure.

How did you rotate figure B?

How far did figure B rotate?

How did the arrow help you?

How did the center help you?

How is using the rotation arrow like using the translation arrow? How is it different?

What do you notice about figure B?

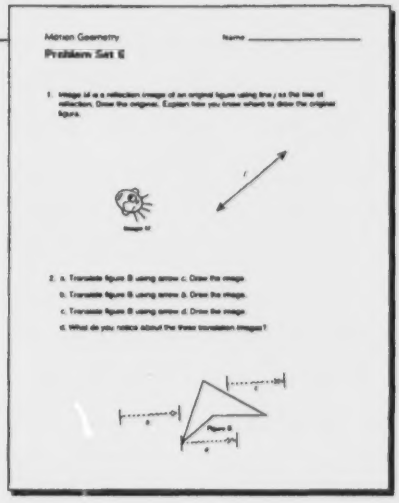
What stayed the same?

What changed?

What determines the location of a rotation image?

Problem Set 6

Multiple Arrows



Problem Set 6, No. 1

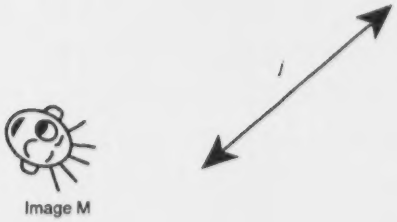
Strand: Reflection

Focus: Reflection image

Task: Extending a line of reflection to find the original figure

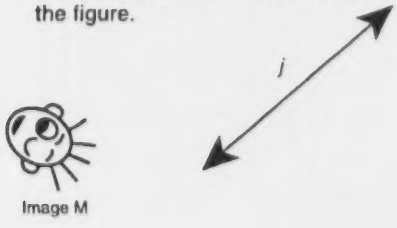
PROBLEM

1. Image M is a reflection image of an original figure using line *j* as the line of reflection. Draw the original. Explain how you knew where to draw the original figure.



ANSWER

1. Explanations will vary but should include a statement about the line of reflection reflecting the figure and the distance between the line and the figure.



Original

Teacher's Insight

This is a reversal problem. The image is given, and the students find the original figure. Students should check that they have been careful with their labels. They should remember that the line of reflection can be extended for construction purposes.

Discussion

How did you find the original figure?

Is the answer unique? Explain.

What determines where a reflection image will be?

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Problem Set 6, No. 2

Strand: Translation

Focus: Translation arrow

Task: Comparing the effects of parallel, congruent translation arrows

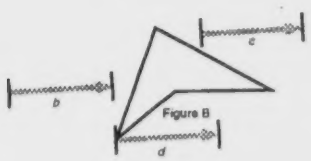
PROBLEM

2. a. Translate figure B using arrow *c*. Draw the image.

   b. Translate figure B using arrow *b*. Draw the image.

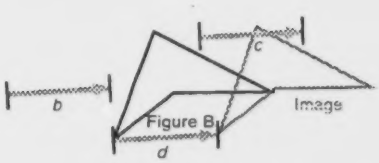
   c. Translate figure B using arrow *d*. Draw the image.

   d. What do you notice about the three translation images?



ANSWER

2. a.–c.



d. The three translation images are congruent and in the same place.

Teacher's Insight

This problem highlights the idea that translation arrows having the same length and pointing in the same direction will result in the same translation. An infinite number of arrows can accomplish the same translation. Observing the effect of using three arrows, all having the same length and all pointing in the same direction, allows students to deepen their understanding of the function of the translation arrow.

If students do each translation using a newly drawn original, they may not see the effect of using congruent arrows. If no one makes the observation, you may want to use tracing paper and overlay the three translations on top of one another.

Discussion

What method did you use to translate the figure?

How far did the image move with each arrow?

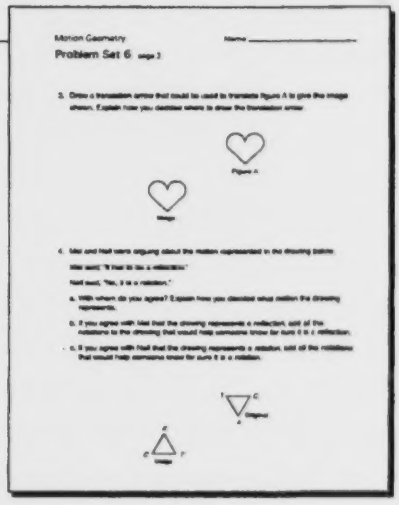
How could you check the distance the figure moved each time?

Why did all the images end up in the same place?

Did you have to do these translations separately? Explain.

How many more arrows could we draw to get the same translation image?

What has to be true for two translation arrows to result in the same image?



Problem Set 6, No. 3

Strand: Translation

Focus: Translation arrow

Task: Creating a translation arrow that gives the indicated result

PROBLEM

3. Draw a translation arrow that could be used to translate figure A to give the image shown. Explain how you decided where to draw the translation arrow.

Figure A

Image

Teacher's Insight

Students usually draw an arrow from an obvious point (such as the point of the heart) on the original to the corresponding point on the image. The discussion should highlight the idea that an arrow connecting any two corresponding points is possible but that some points are easier to identify and connect. Raise the question whether the arrow has to actually touch any point on the figure in order to get the same result. Have students test any arrow that is proposed. Such a test is a good way to correct arrows that were merely sketched and hence may not be the correct direction or the correct length. Students should reiterate the rule that parallel, congruent arrows will give the same image.

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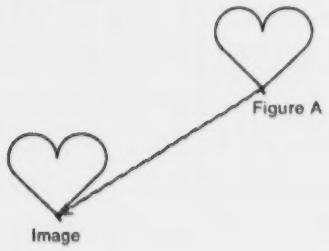
ANSWER

3. Any arrow will work that is in a straight line connecting two corresponding points and is the length of the distance between the two points.



Discussion

Have students share and discuss a variety of answers.

In what ways does a translation arrow help us do the translation?

How did you determine where to draw the arrow?

How did you decide how long to make the arrow?

Is your answer unique?

Was it easier to draw the arrow in some places than in other places?

Is it possible to have an arrow that doesn't touch the figure? Why? Why not?

How many arrows do you need to show how this translation works?

What does a translation arrow tell you about the path of a translation?

How many translation arrows could you possibly draw that would yield the same image?

Describe an arrow that would not yield the same translation image.

Problem Set 6, No. 4

Strand: Rotation

Focus: Properties of a rotation and rotation arrow

Task: Identifying a motion from a drawing of an original figure and its image

PROBLEM

4. Mel and Nell were arguing about the motion represented in the drawing below.

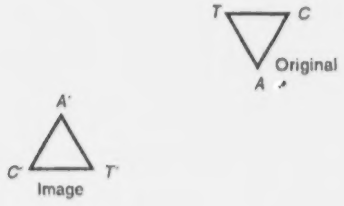
   Mel said, "It has to be a reflection."

   Nell said, "No, it is a rotation."

   a. With whom do you agree? Explain how you decided what motion the drawing represents.

   b. If you agree with Mel that the drawing represents a *reflection*, add all the notation to the drawing that would help someone know for sure it is a reflection.

   c. If you agree with Nell that the drawing represents a *rotation*, add all the notations that would help someone know for sure it is a rotation.



Teacher's Insight

This problem creates arguments for students to analyze. Their different points of view highlight their thinking and some misconceptions that students form as they try to understand new concepts.

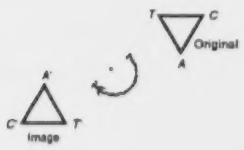
Since a reflection and a 180° rotation seem to look alike, labeling the figures helps students identify the motion. Although a reflection would be possible without the labeling, only a 180° rotation will cause all the points of the original to map onto the corresponding points of the image. Use the opportunity to highlight the function of labeling. Introduce the prime notation ($A'C'T'$) for labeling the vertices of the image. It is another way to indicate an image.

Problem Set 6

ANSWER

4. a. Nell is correct. By comparing the labels at each vertex you can see that *A* and *T* are not in a position to be a reflection. If it were a reflection, the labels would match when the original was mapped over the image. That is not the case here.

c.



Discussion

How did you decide what kind of motion the drawing represented?

How did you test your decision?

Why are the vertices (corners) of the triangles labeled?

How did the labeling help you decide what motion the drawing represents?

What did you add to the drawing?

Why was there confusion in the first place?

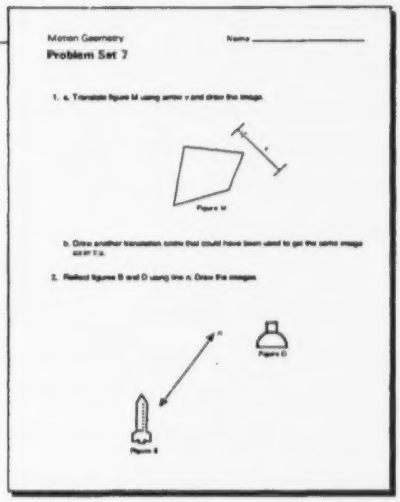
What is the function of labeling in a drawing of a motion?

★HOMEWORK

Problem Set 7

Problem Set 7

Moving the Plane


Motion Geometry
Problem Set 7
Name

Problem Set 7, No.1

Strand: Translation

Focus: Translation arrow

Task: Translating a figure and finding another arrow that will result in the same image

PROBLEM

1. a. Translate figure M using arrow *v* and draw the image.

   b. Draw another translation arrow that could have been used to get the same image as in 1.a.

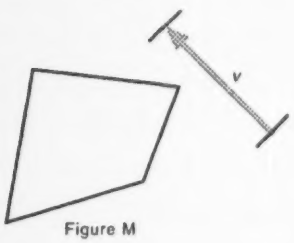


Figure M

ANSWER

1.

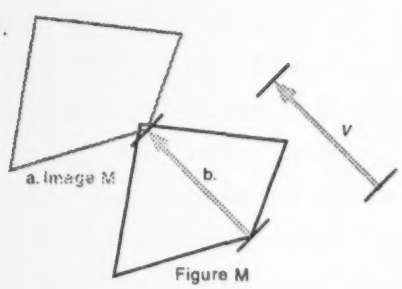


Figure M

b. Any parallel and congruent translation arrow could be used to get the same image.

Teacher's Insight

Students should conclude that parallel congruent translation arrows will yield the same image.

Discussion

Have students share and test a variety of translation arrows.

How did you decide where to draw the second arrow?

How many arrows would yield the same image that you got in 1.a.?

How could you test whether any additional arrow would be a correct possibility?

Why can you use more than one arrow to get the same image?

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Problem Set 7, No. 2

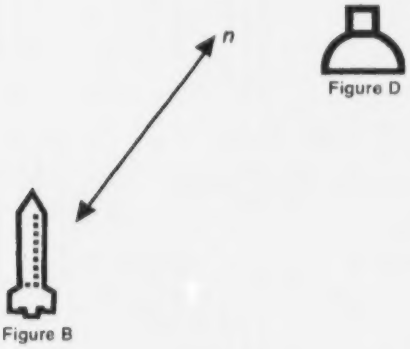
Strand: Reflection

Focus: Line of reflection

Task: Reflecting figures on both sides of the line of reflection, including a figure that is intersected by the line of reflection

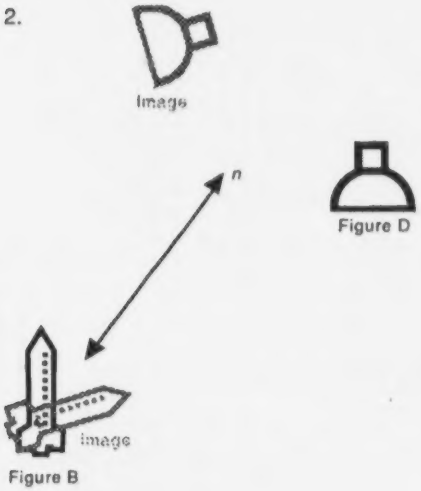
PROBLEM

2. Reflect figures B and D using line *n* and draw the images.



ANSWER

2.



Teacher's Insight

Students may have difficulty reflecting the rocket. It "gets messy" because the line of reflection runs through the figure, making it harder to sort out the parts of the original and the image. With figure B, students should check that they are reflecting on both sides of the line. The original and the image will overlap.

Discussion

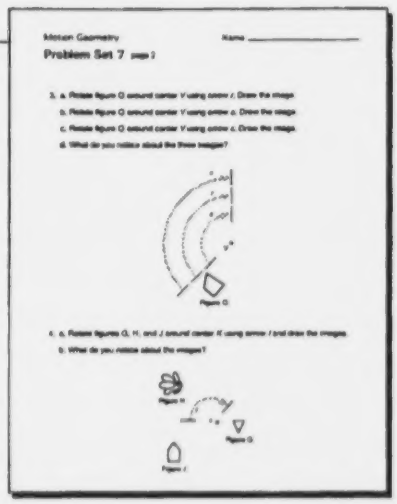
Have students demonstrate how they did the reflections.

What makes this reflection more difficult to do than others?

Did you have to reflect the two figures separately?

Which figure was easier to reflect? Why?

What is your preferred method of reflecting the figure when the line of reflection runs through the original? Explain.



Problem Set 7, No. 3

Strand: Rotation

Focus: Magnitude of a rotation

Task: Comparing the effects of rotation arrows that are of equal arc measure but at different distances from the center of rotation, and concluding that the rotations are congruent

PROBLEM

3. a. Rotate figure Q around center *V* using arrow *r*. Draw the image.

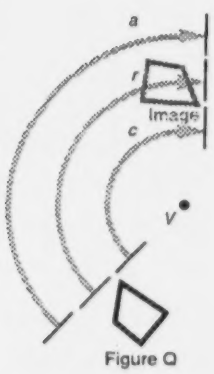
   b. Rotate figure Q around center *V* using arrow *a*. Draw the image.

   c. Rotate figure Q around center *V* using arrow *c*. Draw the image.

   d. What do you notice about the three images?

ANSWER

3. a–c.



d. The three images are identical.

Teacher's Insight

Although the three rotation arrows appear to be of different "lengths," they have the same arc lengths. The arrows indicate rotations that are an equal number of degrees of a circle, and each arrow should result in the same image. The discussion should highlight the distinction between the apparent "length" of the arrow and the portion of the circle it describes.

Discussion

Have students demonstrate and explain how they rotated the figures.

Why did all the images end up in the same place?

How far did figure Q rotate?

Did anything about your results surprise you?

If students are having trouble gauging how far the original will move, ask what tells them how far to rotate. Will the rotation be a full turn? Half a turn? Less than or greater than half a turn?

If students did all rotations separately, ask if they needed to do them separately.

What determines how much to rotate a given figure?

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Problem Set 7, No. 4

Strand: Rotation

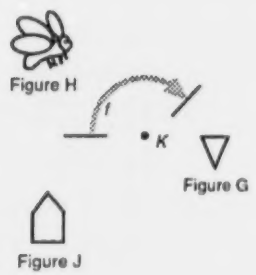
Focus: Properties of a rotation

Task: Judging the effects of rotating multiple figures using the same rotation arrow

PROBLEM

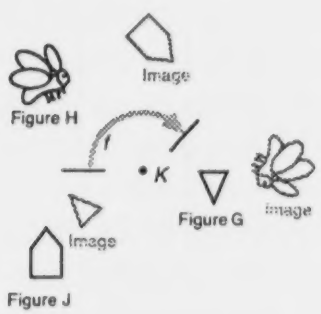
4. a. Rotate figures G, H, and J around center *K* using arrow *f* and draw the images.

   b. What do you notice about the images?



ANSWER

4. a.



b. All the figures can be rotated at the same time. The figures are rotated more than a quarter turn but less than a half turn.

Teacher's Insight

The point of this problem is for students to use their understanding of the individual rotating figures to visualize the rotating plane—the surface the figures lie on.

Discussion

Have students demonstrate and discuss how they rotated the figures to find the images.

How did you do the rotations?

How far did each figure rotate?

How did you decide how much the rotation was?

How can you describe how much each figure rotated?

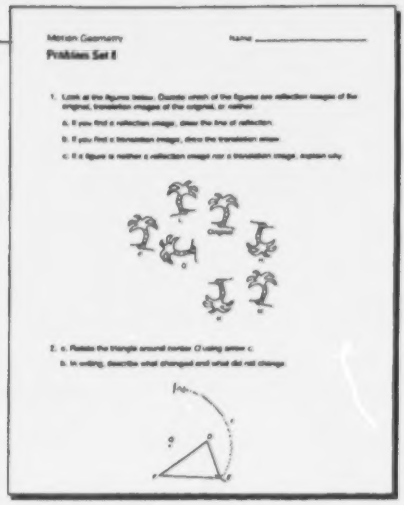
If students talk about all figures moving the same amount, ask why someone might say figure G moved less than figure J.

★ HOMEWORK

Problem Set 8

Problem Set 8

Distinguishing Motions



Problem Set 8, No. 1

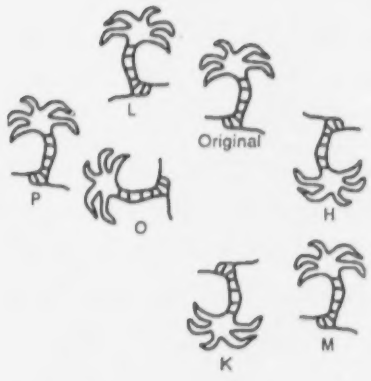
Strand: Reflection and Translation

Focus: Comparing motions

Task: Distinguishing motions of reflections and translations and using a line of reflection or a translation arrow to indicate the motion

PROBLEM

1. Look at the figures below. Decide which of the figures are reflection images of the original, translation images of the original, or neither.
   a. If you find a reflection image, draw the line of reflection.
   b. If you find a translation image, draw the translation arrow.
   c. If a figure is neither a reflection image nor a translation image, explain why.



Teacher's Insight

The problem requires students to identify reflection images and to distinguish them from others. The examples require students to analyze the details of the drawing. For example, they might mistakenly identify figure L as a reflection image because it faces the opposite direction from the original, and they might omit figure O because the line of reflection is at an oblique angle to the figure. Here is a good opportunity for students to test their understanding of the motions. Be sure they remember to draw the lines of reflection and translation arrows.

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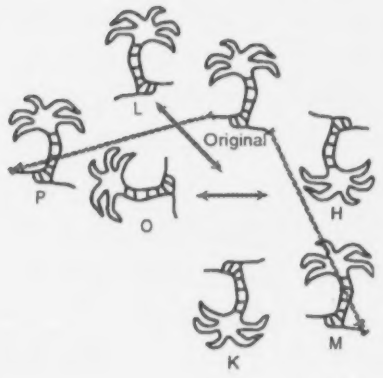
ANSWER

1. a. O and K are reflections.

   b. P and M are translations.

   c. L isn't positioned correctly to be a reflection. L does not map onto the original when the paper is folded. L can't be a translation because it is not facing the same way as the original. H is not a reflection because the paper can't be folded (once) to make H map onto the original. H can't be a translation because it is upside-down from the original.



Discussion

How did you test/decide which motion was used?

How did you know where to draw the line of reflection?

How did you know how to draw the translation arrow?

Were some images more difficult to identify than others? Which ones? Why?

What are some similarities and differences between motions? (You may want to create a definition for a translation and reflection or at least a description of what happens to the original when you do the motion.)

Problem Set 8, No. 2

Strand: Rotation

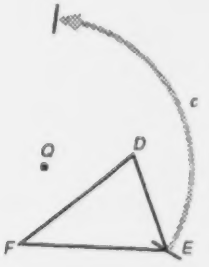
Focus: Properties of a rotation

Task: Rotating a figure and discussing the properties of a rotation

PROBLEM

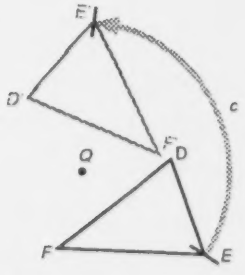
2. a. Rotate the triangle around center Q using arrow c.

   b. In writing, describe what changed and what did not change.



ANSWER

2. a.



b. The triangle now looks upside down. The image is congruent to the original; the size and shape did not change. The center and arrow did not change.

Teacher's Insight

Most of your students should have mastered the skill. Students who are still having difficulty may need extra one-on-one help.

Discussion

How did you use center Q?

How did you use arrow c?

How much did triangle FED turn?

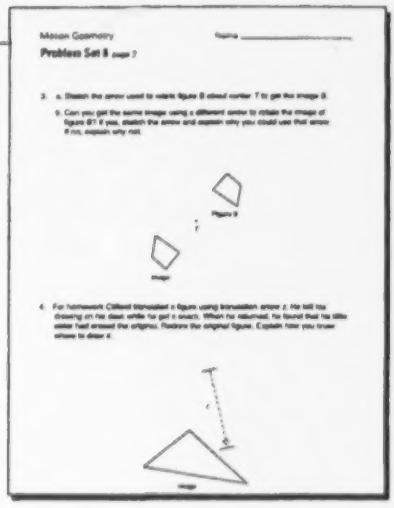
How do you know where to stop?

What changed? What did not change?

What happened to the letters used for labeling? What is their function?

Why do you need to use a center of rotation?

Why do you need to use a rotation arrow?



Problem Set 8, No. 3

Strand: Rotation

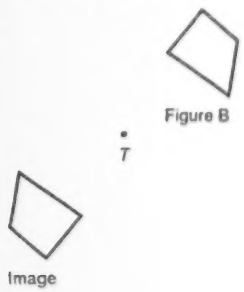
Focus: Rotation arrows

Task: Deciding how far a figure is rotated to result in the given image and drawing a rotation arrow that could have been used; proposing and comparing alternate arrows

PROBLEM

3. a. Sketch the arrow used to rotate figure B about center *T* to get the image.

   b. Can you get the same image using a different arrow to rotate figure B? If yes, sketch the arrow and explain why you could use that arrow. If no, explain why not.



Teacher's Insight

In this problem students test the effect of using arrows of different arc lengths to achieve a given rotation. The arrow can rotate the original figure clockwise or counter-clockwise. For many students such precision is difficult because they have not had enough experience with circles. One technique students use to sketch an arrow is to connect a point on the original figure to a corresponding point on the image. Have students explain how they decided what the arrow should look like. More experienced students may use a compass to draw a more accurate arrow.

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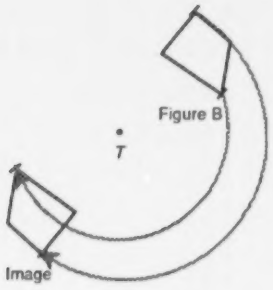
Problem Set 8

ANSWER

3. a.



b. An infinite number of arrows is possible. They should all be 180° and coincide with a half-circle whose center is the center of rotation.

Discussion

Have students share and test a variety of rotation arrows.

How did you create an arrow that you could use to rotate the figure?

What did the arrow have to do to work in this rotation?

How did you decide how much of the circle the arrow had to cover?

How did you decide on the direction of your arrow?

How could you test your sketch?

What other arrows could give you the same results?

What shape is your arrow? Why is it that shape?

How could you make your arrow more accurate?

Problem Set 8, No. 4

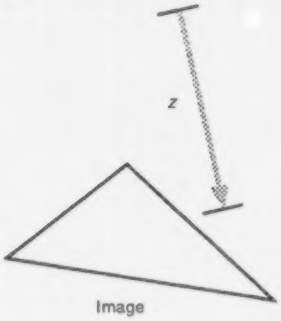
Strand: Translation

Focus: Translation image

Task: Finding the original figure using a translation image and arrow

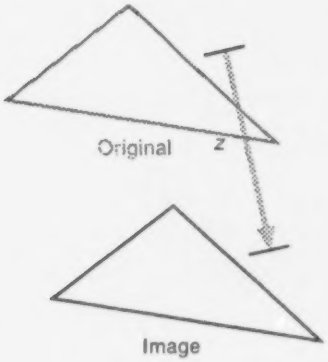
PROBLEM

4. For homework Clifford translated a figure using translation arrow *z*. He left his drawing on his desk while he got a snack. When he returned, he found that his little sister had erased the original. Redraw the original figure. Explain how you knew where to draw it.



ANSWER

4.



Teacher's Insight

Reversal problems using translations tend to be more difficult for students than reversal problems using reflections. This is because the arrow's indication of direction can be distracting. If students have difficulty deciding where the original figure should be, have them focus on the arrow and make a prediction. You may need to help students understand the problem situation by stressing that the drawing represents what happened *after* a figure was translated using the given arrow. They need to find where the figure was *before* the motion occurred.

Discussion

Have students demonstrate how they used the image and the arrow to find the original figure.

How did you decide where to draw the original?

How did the arrow help you find the original?

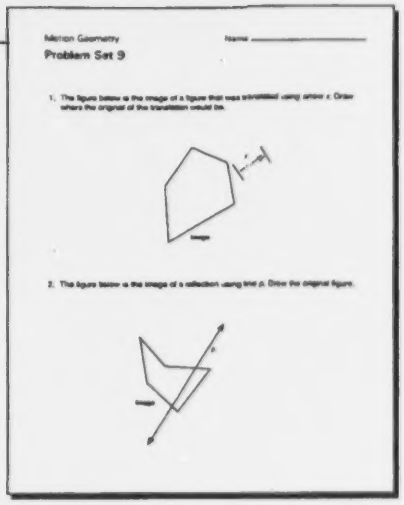
Why is the original in the opposite direction of the arrow?

★HOMEWORK

Problem Set 9

Problem Set 9

Finding the Original


Motion Geometry Name _____
Problem Set 9
1. The figure below is the image of a figure that was translated using arrow x. Draw where the original of the translation would be.
2. The figure below is the image of a reflection using line ℓ. Draw the original figure.

Problem Set 9, No. 1

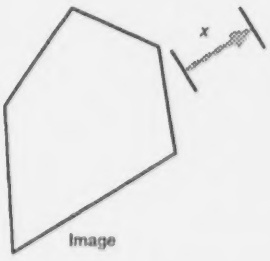
Strand: Translation

Focus: Translation image

Task: Finding an original figure that overlaps the translated image

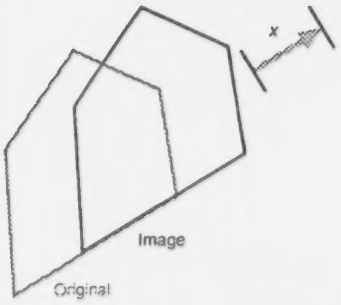
PROBLEM

1. The figure below is the image of a figure that was translated using arrow *x*. Draw where the original of the translation would be.



ANSWER

1.



Teacher's Insight

This problem gives students more practice with reversing translations. Require students to test their answers and explain how they decided where the original figure would be. Students generally use one of two tests: (1) reverse the direction of the arrow to do a translation from the image to the original; (2) translate the original you find and see if it maps onto the given image.

Discussion

Have students demonstrate how they moved the figure to get the original.

What do we need to know to be able to re-create a translation?

How did you decide where to draw the original?

Is your answer unique? Explain.

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Problem Set 9, No. 2

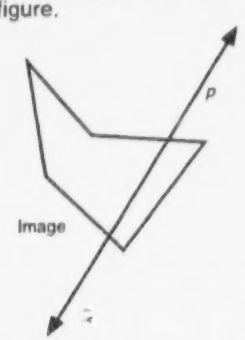
Strand: Reflection

Focus: Line of reflection

Task: Finding the original figure using a reflection image and a line of reflection

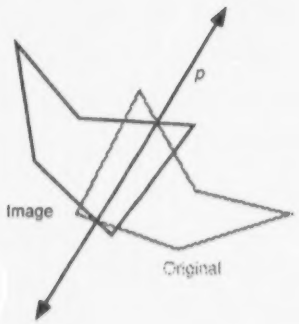
PROBLEM

2. The figure below is the image of a reflection using line *p*. Draw the original figure.



ANSWER

2.



Teacher's Insight

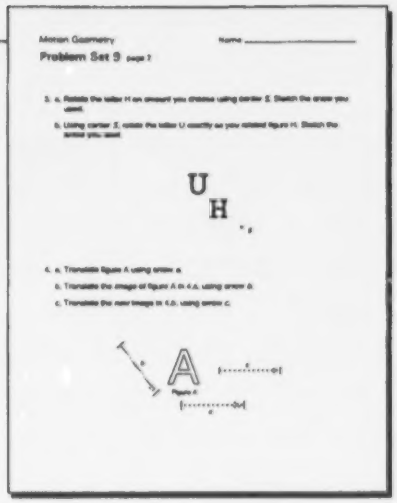
Be sure students draw the original on both sides of the reflection line in one picture. Careful labeling helps students keep track of the original and image. Some students use different color pens to distinguish between the original and image drawings.

Discussion

How did you know where to draw the original figure?

Is your answer unique? Explain.

Why do the original and image overlap?



Problem Set 9, No. 3

Strand: Rotation

Focus: Magnitude of a rotation

Task: Creating congruent rotations and comparing arc measures to arc lengths

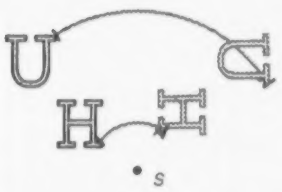
PROBLEM

3. a. Rotate the letter H an amount you choose using center *S*. Sketch the arrow you used.

   b. Using center *S*, rotate the letter U exactly as you rotated figure H. Sketch the arrow you used.

ANSWER

3. a–b. Multiple answers are possible and they may include the following:



Teacher's Insight

Students decide how far they want to rotate one figure and then try to create a congruent rotation for the other figure. In 3.b. some students realize that they can achieve a congruent rotation for figure U by using the same arrow they sketched in 3.a. Students also find out that the more accurate their sketches are, the better the results in 3.b.

Discussion

Have students share and discuss a variety of answers.

How did you decide how much to rotate figure H?

How did you draw the arrow for that rotation?

How did you rotate figure U the same amount?

How did you decide it was the same amount?

How did you sketch the arrow for that rotation?

Did you have to draw a separate arrow for the rotation in 3.b.?

Did one figure rotate or move "farther" than the other? Explain your thinking.

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Problem Set 9, No. 4

Strand: Translation

Focus: Multiple translations

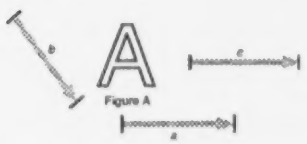
Task: Doing a series of translations

PROBLEM

4. a. Translate figure A using arrow *a*.

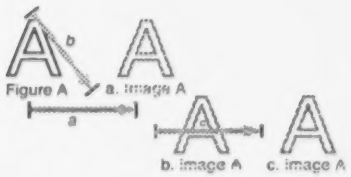
   b. Translate the *image* of figure A in 4.a. using arrow *b*.

   c. Translate the *new* image in 4.b. using arrow *c*.



ANSWER

4. a–c.



★HOMEWORK

Problem Set 10

Teacher's Insight

This is the first time students do more than one motion, in this case, a series of translations. This is a good problem to help students make the connection between transformations and measurement. Multiple translations are like iterations of a measurement unit. Have students follow the problem instructions carefully.

Discussion

How was this problem different from other translation problems you've done?

How many motions did you use in this problem?

What happened to the images?

What does the image from 4.a. become in 4.b? What does the image from 4.b. become in 4.c.?

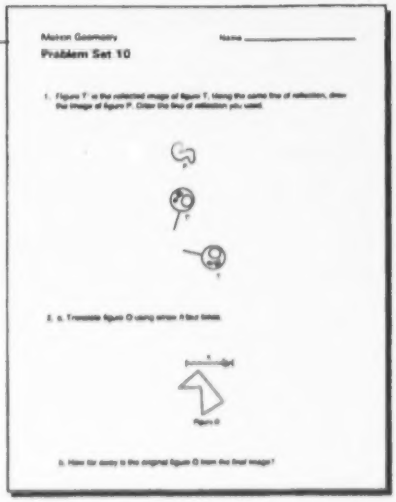
What determined where you drew the images?

What do you notice about arrows *a* and *c*?

What was the same/different about the motions in each part of the problem?

Problem Set 10

Reproducing Motions


Motion Geometry Name _____

Problem Set 10

1. Figure T′ is the reflected image of figure T. Using the same line of reflection, draw the image of figure P. Draw the line of reflection you used.

Problem Set 10, No. 1

Strand: Reflection

Focus: Finding the line of reflection

Task: Finding the line of reflection using the image and original, and using the line to reflect a second figure

PROBLEM

1. Figure T′ is the reflected image of figure T. Using the same line of reflection, draw the image of figure P. Draw the line of reflection you used.

Teacher's Insight

This problem gives students the opportunity to review and solidify the concepts of a reflection. You may want to preview this problem with students to be sure they can identify the original and the image. Students sometimes talk about the line of reflection as a line of symmetry. If they do, ask them to explain how they are using the term "symmetry." Ask them how they know the originals and their images are identical.

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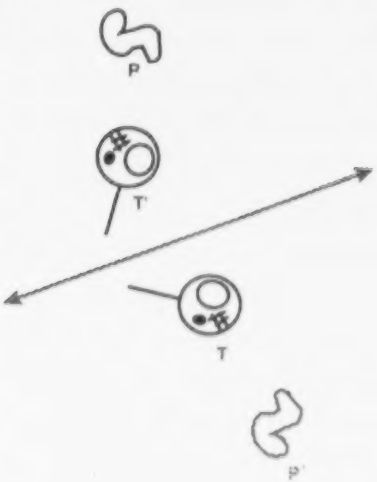
ANSWER



Discussion

How did you find the line of reflection?

How did you create the image of figure P?

Are any other lines of reflection possible? Why or why not?

Could you have found any other images of figure P using the same line of reflection? Explain.

Problem Set 10, No. 2

Strand: Translation

Focus: Translation of a translation

Task: Relating measurement to transformations by showing how doing multiple translations using the same arrow is like iterating a unit

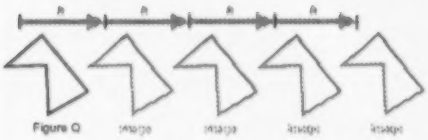
PROBLEM

2. a. Translate figure Q using arrow *h* four times.

b. How far away is the original figure Q from the final image?

ANSWER

2. a.



b. The original is four arrow lengths (four *h* lengths) from the final image.

Teacher's Insight

This is another problem that helps students understand the connection between transformations and measurement. The question of how far the original figure is from the final image leads students to examine the concept of a unit of measurement. The final image is four arrow lengths away from the original figure. Students often try to measure the distance using a standard unit of measurement, but since no unit is specified, the only way they can agree on a length is to use the arrow as the unit of measure. Check also how students interpret using the translation arrow four times. They should actually move the original figure and trace the image four times.

Discussion

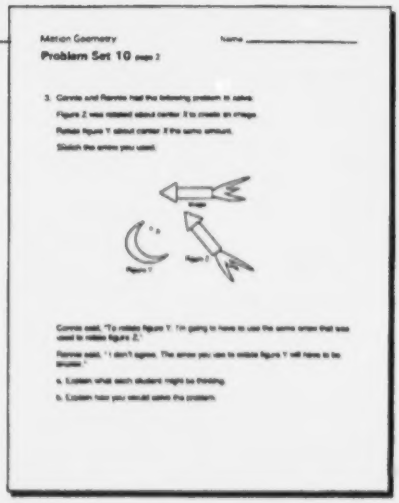
Have students demonstrate and discuss how they translated figure Q.

How did you decide when to stop doing translations?

How did you decide how far away the original figure was from the final image?

If you did the translation two more times, how far away would the final image be from the original?

How many times would you have to translate the figure for the final image to be 15 arrow lengths away from the original?



Problem Set 10, No. 3

Strand: Rotation

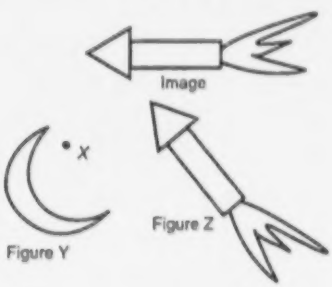
Focus: Arc measure of a rotation arrow

Task: Discussing the relationship between an object's distance from the center of rotation and the arc measure of the rotation arrow needed to rotate it to a given position

PROBLEM

3. Connie and Ronnie had the following problem to solve:

   Figure Z was rotated about center *X* to create an image.
   Rotate figure Y about center *X* the same amount.
   Sketch the arrow you used.



   Connie said, "I'm going to have to use the same arrow to rotate figure Y that I used to rotate figure Z."

   Ronnie said, "I don't agree. The arrow you use to rotate figure Y will have to be shorter."

   a. Explain what each student might be thinking.

   b. Explain how you would solve the problem.

Teacher's Insight

Figures located farther from the center of rotation seem to rotate farther because they need to move the same arc length on a larger circle than figures located closer to the center. This problem is designed to help students clarify their thinking about the arc length of rotation arrows and emphasizes the concept of rotating the plane. The discussion should lead students to conclude that any rotation arrow having the same center and arc length should result in the same image. (See Problem Set 7, No.3, for reference.)

This is a relatively wordy problem; you may want to preview it.

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Problem Set 10

ANSWER

3. a Connie thinks that in order to rotate something the same amount, she must use the same rotation arrow. Ronnie thinks that because figure Y is closer to the center of rotation, the figure is making a smaller rotation around the center. Because figure Y doesn't need to move far to go completely around the center, Ronnie thinks that the arrow he uses has to be shorter.

   b. First you need to find the arrow that was used to rotate figure Z. Then trace the arrow, the center, and figure Y. Rotate the paper in the direction of the arrow while holding a point fixed on the center. Finally trace the position of the image.

Discussion

How would you solve the problem?

What could Connie mean by the "same" arrow?

What could Ronnie mean by a "shorter" arrow?

How does the rotation arrow affect the rotation?

How does the center of rotation affect the rotation?

How do you determine the "amount" of a rotation?

Did you test your solution? How?

Reshaping Mathematics for Understanding

Motion Geometry

Blackline Masters

Motion Geometry Name _____

Lab A

1. Choose an object from the collection you are given.

2. a. Trace the object on your paper.

   b. Move it to a different place on your paper and trace it again.

   c. Make a drawing to show how you moved the object.

3. a. Trace the object again on a second sheet of paper.

   b. Move it to a different place on your paper using a different motion from the one you used in 2.a. Trace it.

   c. Make a drawing to show how you moved the object.

4. a. Trace the object again on a third sheet of paper.

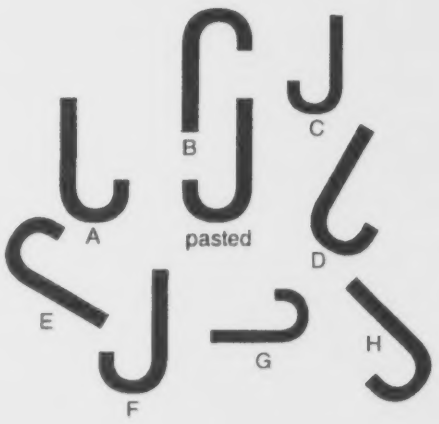
   b. *Without lifting it,* move your object to a different place on your paper. Trace where it ends up.

   c. Make a drawing to show how you moved the object.

5. What information did you include in your three drawings?

Now try the following problem:

6. Carrie was pasting some letter *J*s on a poster. She had pasted only one when a gust of wind scattered the others. Describe clearly the position of each letter *J* in relation to the one pasted on the poster.



Motion Geometry Name _____

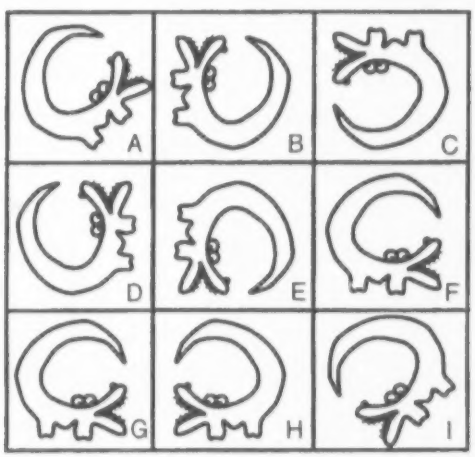
Problem Set 1

1. Kelly sees a bird soaring over a still, smooth lake. She stands at the edge of the lake and is confused because she also sees what looks like the same bird in the lake.

   a. Draw what Kelly sees in the water.

   b. Compare the bird with its reflection in the water. What is the same about the bird and its reflection? What is different about them?

2. Casey's auntie sent him some alligator chocolates from her candy factory. The pieces got shaken up in the mail, and when Casey opened the box, this is what he found. Describe what happened to each piece of candy.



Name _____

Problem Set 1 page 2

3. Look straight into the mirror.

   a. Draw your face.

   b. Close one eye. Draw your face again.

   c. Hold up your outspread hand with the palm facing the mirror. Draw your hand.

4. Write about your drawings.

   a. How are your drawings the same as your actual body?

   b. How are the drawings different from your actual body?

   c. What is special about a reflection?

Name _____

Lab B

1. a. Kaleo made a drawing showing figure A and its reflection. Explain how he might have gotten the reflection.

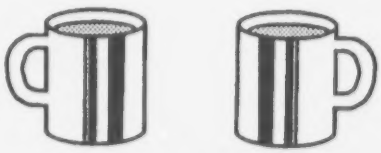


Figure A Reflection

b. Describe everything you notice about figure A and its reflection.

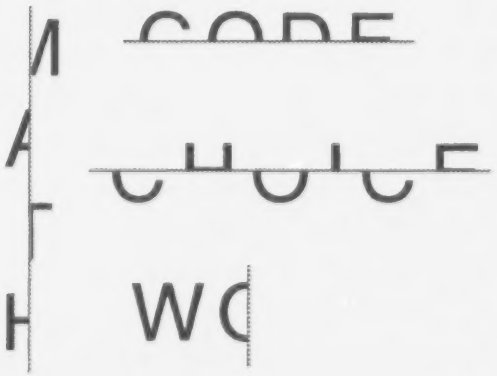
c. Trace figure A on your paper.

d. Using your traced figure, do a reflection that differs from the one Kaleo did.

e. Explain how you created your reflection.

f. Compare the reflection you did with the one Kaleo did. What is the same about them? What is different?

2. Tim and some friends formed a club in which they sent each other words with hidden parts.



a. What are these mystery words? Explain how you found your answers.

b. Write at least three other words that Tim and his friends could send each other using this system.

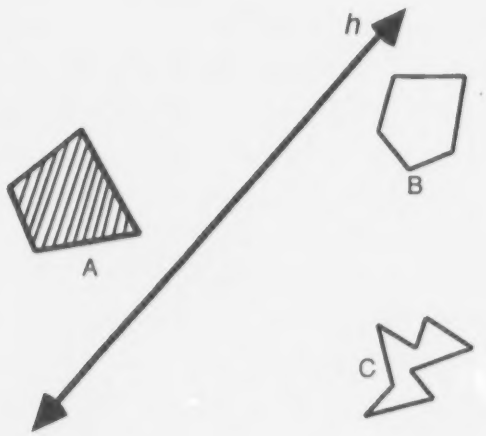
Name _____

Problem Set 2

1. a. Predict where figure A will end up if you reflect it over the line. Mark your prediction with an *a*. Explain how you made your prediction.

   b. Predict where figure B will end up if you reflect it over the line. Mark your prediction with a *b*. Explain how you made your prediction.

   c. Predict where figure C will end up if you reflect it over the line. Mark your prediction with a *c*. Explain how you made your prediction.



2. a. Draw a reflection of figure B.

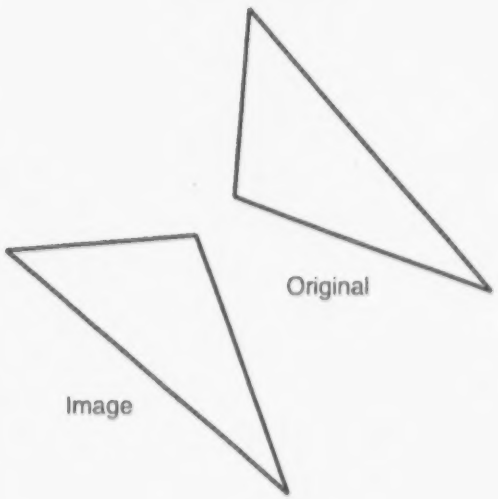
   b. Draw a different reflection of figure B.

   c. Draw yet a different reflection of figure B.

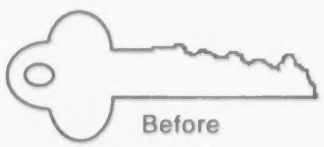
Figure B

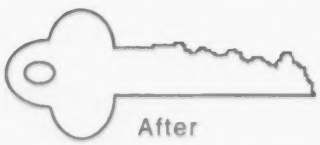
Problem Set 2 page 2

3. Sherri reflected a triangle as shown below, but she forgot to draw the line of reflection. Find the line of reflection she used and draw it. Explain how you decided where to draw the line.



4. Christopher was working on a lab about motions. He traced a key and then slid it. He drew the picture below to represent his motion. What could Christopher have added to his drawing to show how he slid the key?



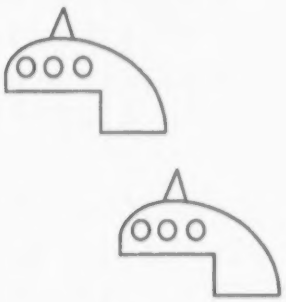


Name _____

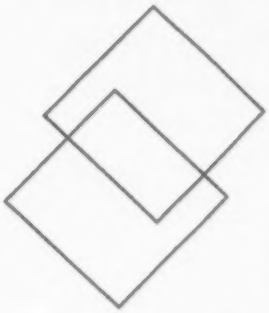
Lab C

1. Shannon traced a couple of objects, slid them, then traced their images. Add markings to Shannon's drawings to show how she might have done the slides.

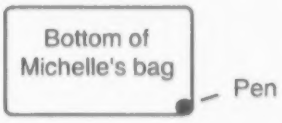
a.



b.



2. Michelle slid her bag on the dining room floor.



She didn't notice that a pen was sticking out of a hole at the bottom of her bag. The pen left a mark on the floor.

Pen mark

Draw the position of the bottom of Michelle's bag before and after it made the mark on the floor.

3. The drawing below shows the image of figure X after it was slid.

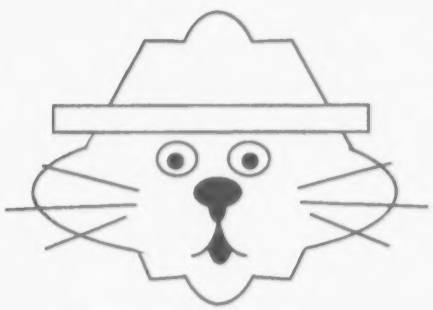
Image

a. Draw the original. Be sure to indicate in your drawing how figure X was slid.

b. Explain how you decided where to draw the original.

4. a. Use line *l* to reflect the cat.

b. Explain how you can check to see that your reflection is correct.



Original

Name _____

Problem Set 3

1. a. Draw a line of reflection and use it to reflect the two figures.

   b. Without actually reflecting the figure, draw a line of reflection so that the image of the duck would face left. Explain how you decided where to draw the line.

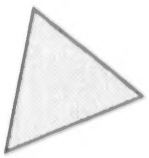
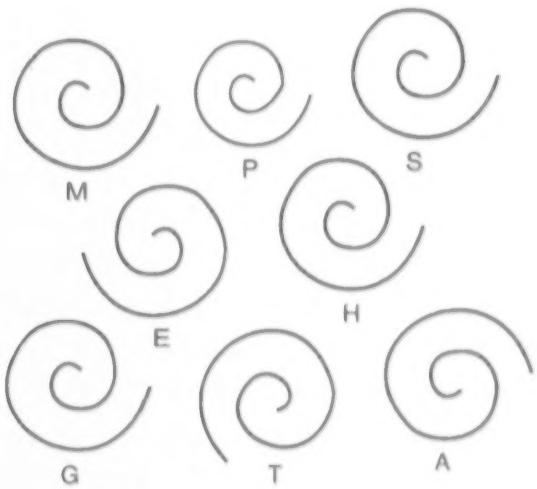


Figure P

Figure W

2. a. Which of these figures when translated could be an image of figure H?

   b. Explain how you made your selection(s).

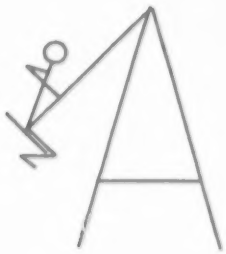


Name _____

Problem Set 3 page 2

3. Gerald loves the swings at the playground. He is not afraid to go very high. The drawing below shows Gerald getting ready to swing forward.

   Draw a picture of Gerald that shows how high he swings and the path of his swing.



4. a. Draw the line of reflection used to reflect figures C and D. Explain how you found the line.

   b. Use the reflection line you found in 4.a. to reflect figure B.

Figure B

Figure C

Image of C

Figure D

Image of D

Name _____

Problem Set 4

1. a. Draw a line of reflection and use it to reflect figure P.

   b. Draw a different line of reflection so that figure P's image is as far away from the original as possible.

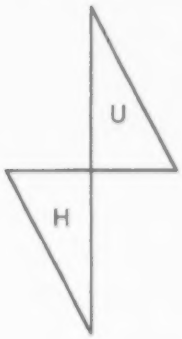
   c. Draw another line of reflection so that the image is as close to the original as possible.

Figure P

2. Travis's new watch has a picture of a boat on the minute hand. At 10:10 it looks like the picture below. Draw a picture to show how the minute hand will look at 10:25.

Problem Set 4 page 2

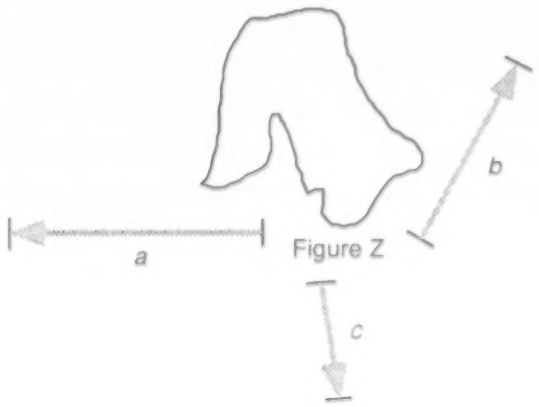
3. Rotate triangle U to completely cover triangle H. On your drawing, show how you rotated triangle U.



4. a. Predict where the image will end up if you slide figure Z using arrow *a*. Mark your prediction with an A. Explain how you made your prediction.

   b. Predict where the image will end up if you slide figure Z using arrow *b*. Mark your prediction with a B. Explain how you made your prediction.

   c. Predict where the image will end up if you slide figure Z using arrow *c*. Mark your prediction with a C. Explain how you made your prediction.

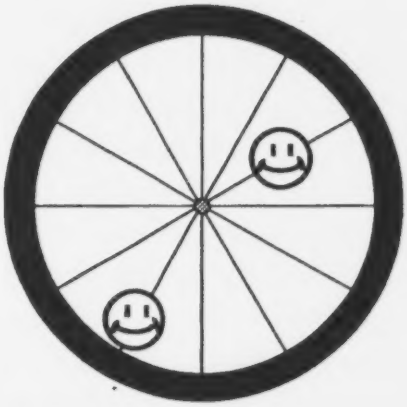


Motion Geometry Name _____

Lab D

1. Peter turned his bycicle upside down so he could attach picture clips to the spokes of the wheel.

   a. Draw what the bicycle wheel would look like if it were rotated a quarter ($\frac{1}{4}$) turn to the left.



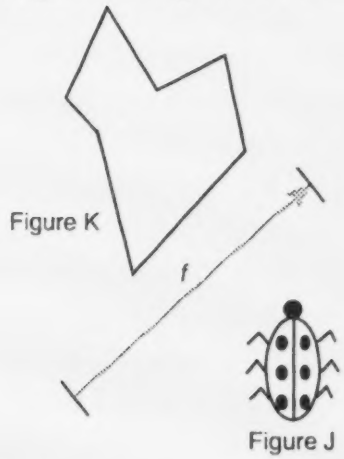
   b. Describe how the faces moved.

   c. Explain how the faces changed, and how they did not change.

   d. Find the center of the wheel. Did the center move to a new location?

   e. What would the wheel look like if it were rotated a full turn?

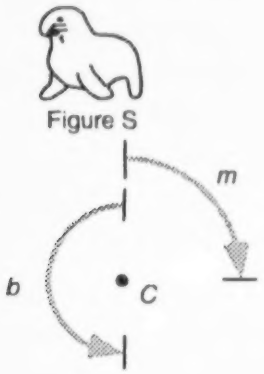
2. Translate figure K and figure J using arrow *f* and draw the images.



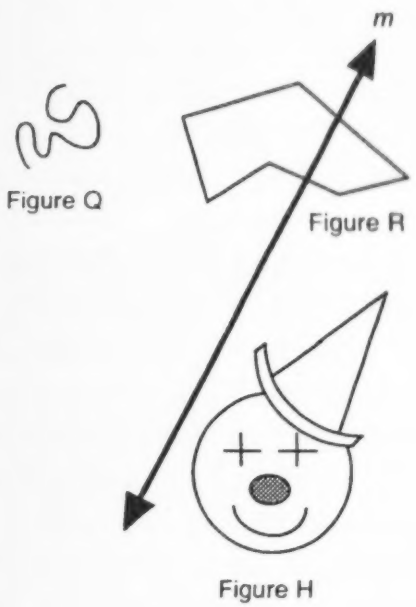
Lab D page 2

3. a. Predict where the image would end up if you rotate figure S around center *C* using arrow *m*. Label your prediction L. Explain how you made your prediction.

   b. Predict where the image would end up if you rotate figure S around center *C* using arrow *b*. Label your prediction E. Explain how you made your prediction.



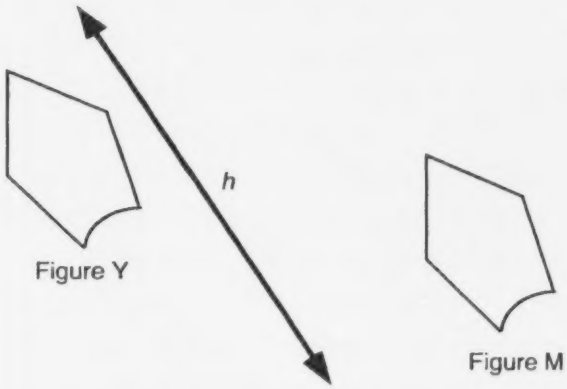
4. Reflect the figures over line *m* and draw the images.



Name _____

Problem Set 5

1. Reflect figure Y and figure M over line *h*. Compare the two images. What is the same about them? What is different about them?

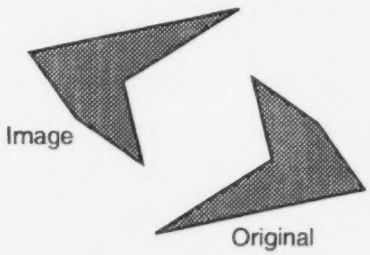


2. Malia did a motion on a given figure. The drawing below represents the figure and its image after she did the motion.

   a. Describe the motion Malia used.

   b. Test your description by doing the motion. You should get the same image as Malia did.

   c. Make a sketch to represent your test.



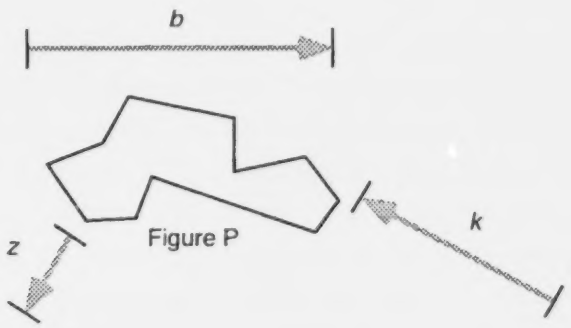
Name _____

Problem Set 5 page 2

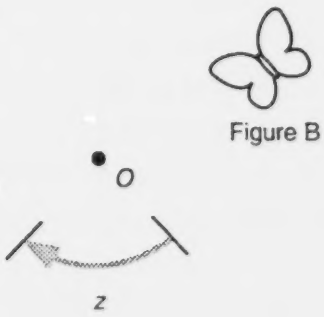
3. a. Translate figure P using arrow *k*. Draw the image.

   b. Translate figure P using arrow *b*. Draw the image.

   c. Translate figure P using arrow *z*. Draw the image.



4. Rotate figure B around center *O* using arrow *z*. Draw the image.

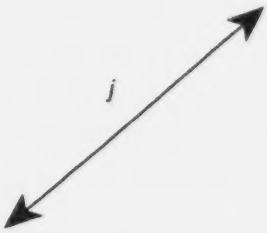


Motion Geometry Name _____

Problem Set 6

1. Image M is a reflection image of an original figure using line *j* as the line of reflection. Draw the original. Explain how you knew where to draw the original figure.

Image M

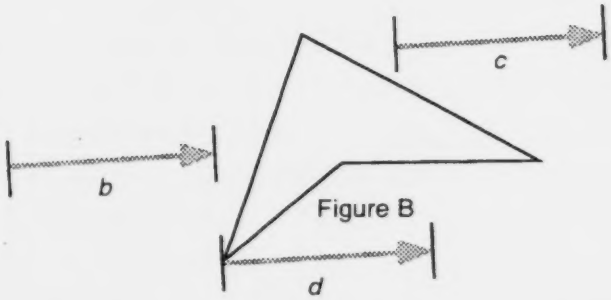


2. a. Translate figure B using arrow *c*. Draw the image.

   b. Translate figure B using arrow *b*. Draw the image.

   c. Translate figure B using arrow *d*. Draw the image.

   d. What do you notice about the three translation images?



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Problem Set 6 page 2

3. Draw a translation arrow that could be used to translate figure A to give the image shown. Explain how you decided where to draw the translation arrow.

Figure A

Image

4. Mel and Nell were arguing about the motion represented in the drawing below.

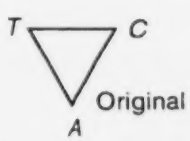
   Mel said, "It has to be a reflection."

   Nell said, "No, it is a rotation."

   a. With whom do you agree? Explain how you decided what motion the drawing represents.

   b. If you agree with Mel that the drawing represents a *reflection*, add all the notations to the drawing that would help someone know for sure it is a reflection.

   c. If you agree with Nell that the drawing represents a *rotation*, add all the notations that would help someone know for sure it is a rotation.



Name _____

Problem Set 7

1. a. Translate figure M using arrow *v* and draw the image.

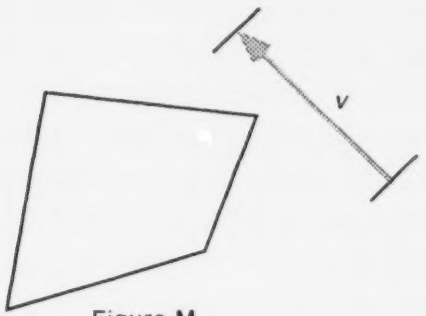


Figure M

b. Draw another translation arrow that could have been used to get the same image as in 1.a.

2. Reflect figures B and D using line *n*. Draw the images.

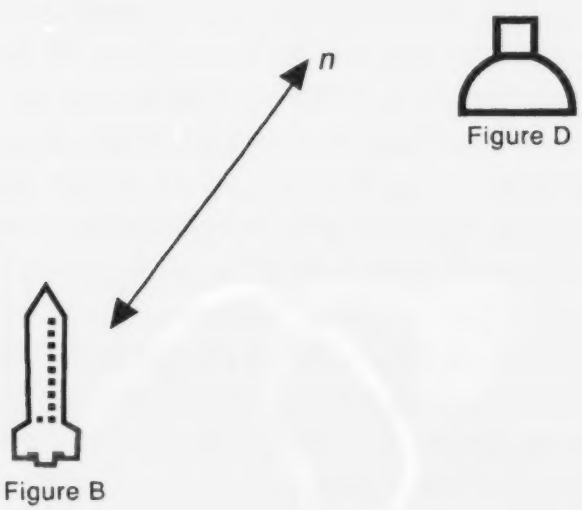


Figure D

Figure B

Problem Set 7 page 2

3. a. Rotate figure Q around center *V* using arrow *r*. Draw the image.

   b. Rotate figure Q around center *V* using arrow *a*. Draw the image.

   c. Rotate figure Q around center *V* using arrow *c*. Draw the image.

   d. What do you notice about the three images?

Figure Q

4. a. Rotate figures G, H, and J around center *K* using arrow *f* and draw the images.

   b. What do you notice about the images?

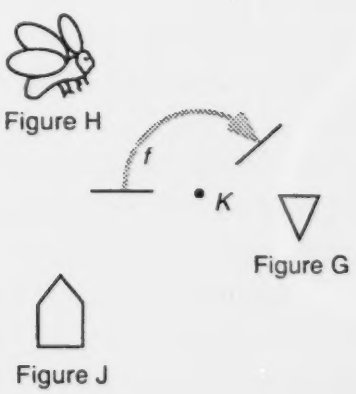


Figure H

Figure G

Figure J

Name _____

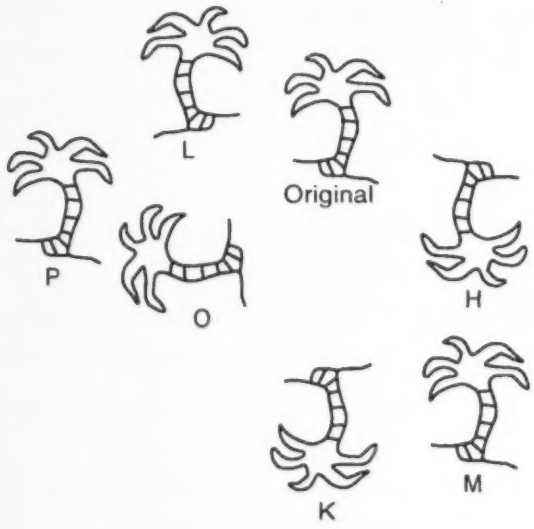
Problem Set 8

1. Look at the figures below. Decide which of the figures are reflection images of the original, translation images of the original, or neither.

   a. If you find a reflection image, draw the line of reflection.

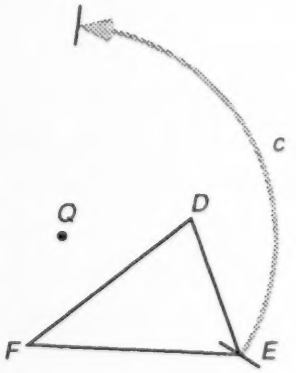
   b. If you find a translation image, draw the translation arrow.

   c. If a figure is neither a reflection image nor a translation image, explain why.



2. a. Rotate the triangle around center *Q* using arrow *c*.

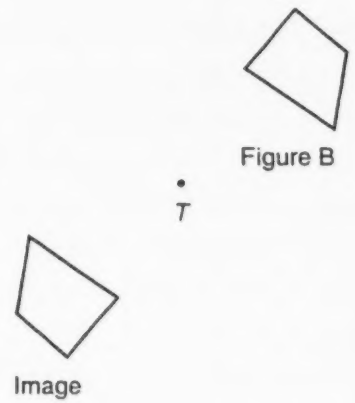
   b. In writing, describe what changed and what did not change.



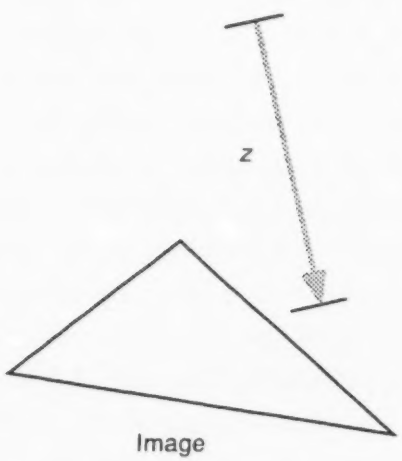
Problem Set 8 page 2

3. a. Sketch the arrow used to rotate figure B about center *T* to get the image.

   b. Can you get the same image using a different arrow to rotate figure B? If yes, sketch the arrow and explain why you could use that arrow. If no, explain why not.



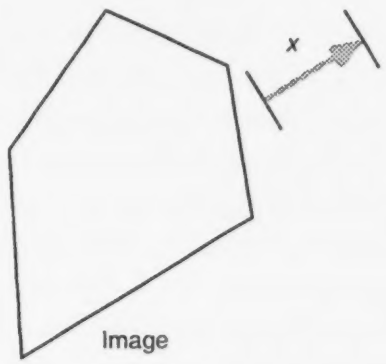
4. For homework Clifford translated a figure using translation arrow *z*. He left his drawing on his desk while he got a snack. When he returned, he found that his little sister had erased the original. Redraw the original figure. Explain how you knew where to draw it.



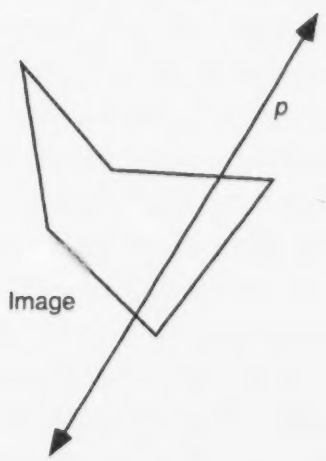
Name _____

Problem Set 9

1. The figure below is the image of a figure that was translated using arrow *x*. Draw where the original of the translation would be.



2. The figure below is the image of a reflection using line *p*. Draw the original figure.

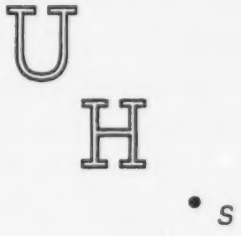


Name _____

Problem Set 9 page 2

3. a. Rotate the letter H an amount you choose using center *S*. Sketch the arrow you used.

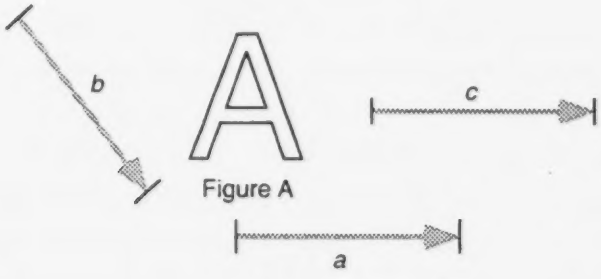
   b. Using center *S*, rotate the letter U exactly as you rotated figure H. Sketch the arrow you used.



4. a. Translate figure A using arrow *a*.

   b. Translate the *image* of figure A in 4.a. using arrow *b*.

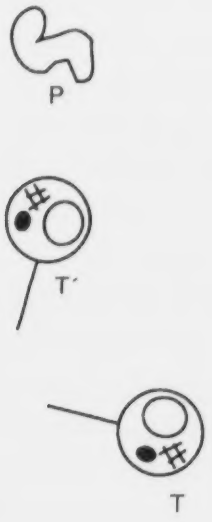
   c. Translate the *new* image in 4.b. using arrow *c*.



Motion Geometry Name _____

Problem Set 10

1. Figure T′ is the reflected image of figure T. Using the same line of reflection, draw the image of figure P. Draw the line of reflection you used.



2. a. Translate figure Q using arrow *h* four times.

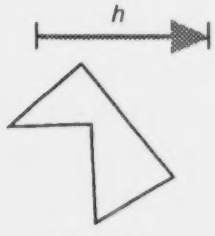


Figure Q

b. How far away is the original figure Q from the final image?

Name _____

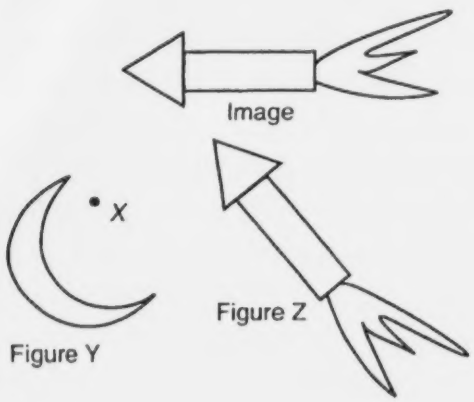
Problem Set 10 page 2

3. Connie and Ronnie had the following problem to solve:

   Figure Z was rotated about center *X* to create an image.

   Rotate figure Y about center *X* the same amount.

   Sketch the arrow you used.



Connie said, "To rotate figure Y, I'm going to have to use the same arrow that was used to rotate figure Z."

Ronnie said, " I don't agree. The arrow you use to rotate figure Y will have to be shorter."

a. Explain what each student might be thinking.

b. Explain how you would solve the problem.

Reshaping Mathematics for Understanding

Motion Geometry

Resources

Motion Geometry

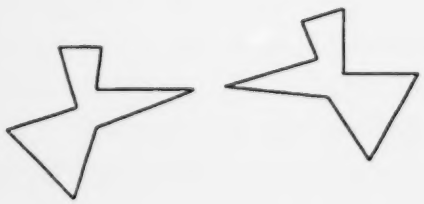
Sample Assessment Problems

1. For his homework one night, Matt had to do three motions and draw the originals and images: a reflection, a translation, and a rotation. He did each motion and drew the images. But then the telephone rang. It was his friend, Pat. They talked on the phone so long that by the time Matt got back to his homework, he forgot which drawing was which.

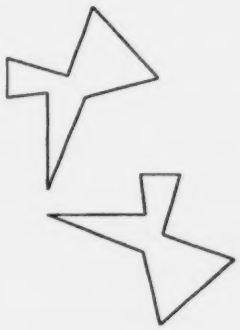
   Look at Matt's drawings below. Copy them onto your paper. Identify the motion in each drawing and add the notations that Matt should include in his drawings so that everyone can tell which motion was used. Explain how you decided which drawing represented which motion.

   a. Drawing 1

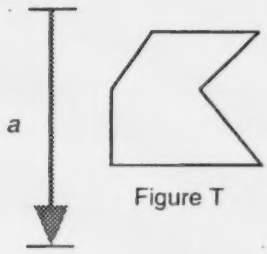
   b. Drawing 2



c. Drawing 3



2. Translate figure T using arrow *a*.



3. a. Use a motion to check if figure E is a square. Explain your thinking.

   b. Use a motion to check if figure F is a square. Explain your thinking.

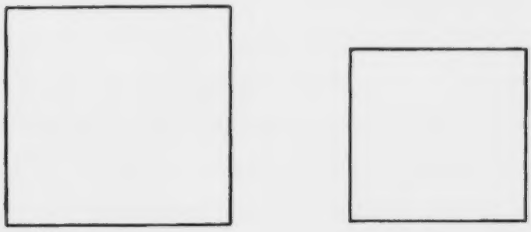
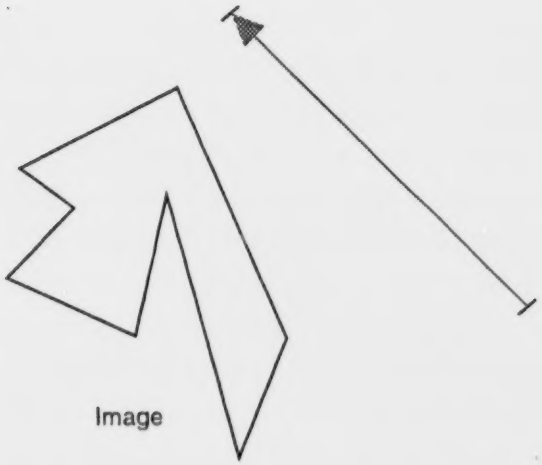


Figure E Figure F

4. Below is an image of a figure after it was translated using the given arrow.

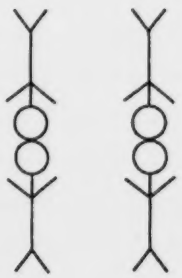
   a. Draw the *original* figure.

   b. Could a different translation arrow have been used to get the same *original* and *image*? If yes, draw it. If no, explain why not.



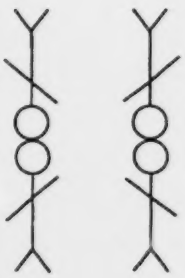
5. Look at each pair of drawings. Decide if each pair represents a reflection, a translation, or a rotation. Explain how you decided. For each pair, if more than one motion is possible, explain every one.

a.



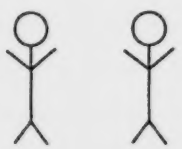
Original Image

b.



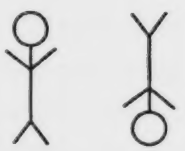
Original Image

c.



Original Image

d.

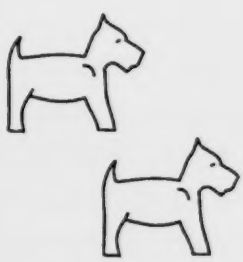


Original Image

6. Look at each pair of drawings (A, B, C). Decide if the pair represents a reflection, a translation, or a rotation. Explain briefly how you decided.

Pair A

Pair B

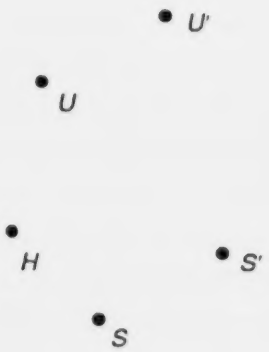


Pair C

7. Points U' and S' are the images of points U and S.

   a. What motion could you use to get the images?

   b. Draw the image of point H using the same motion you described in part a.



8. Below are figure B and its reflection image. Find the line of reflection used to create this image.

Figure B

Image

9. Rotate figure E using arrow *d* and center *W*.

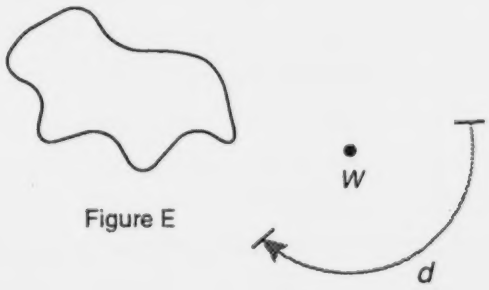
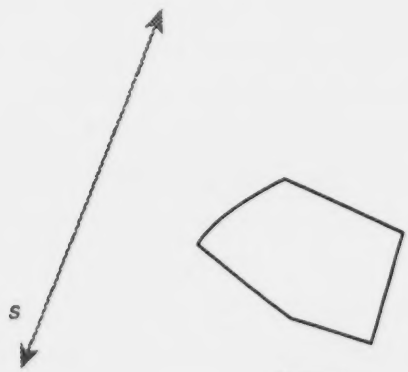


Figure E

10. Below is an image of a figure after it was reflected using line *s*.

    a. Draw the original.

    b. Could a different line of reflection be used to get the same original and image? If yes, draw it. If no, explain why not.



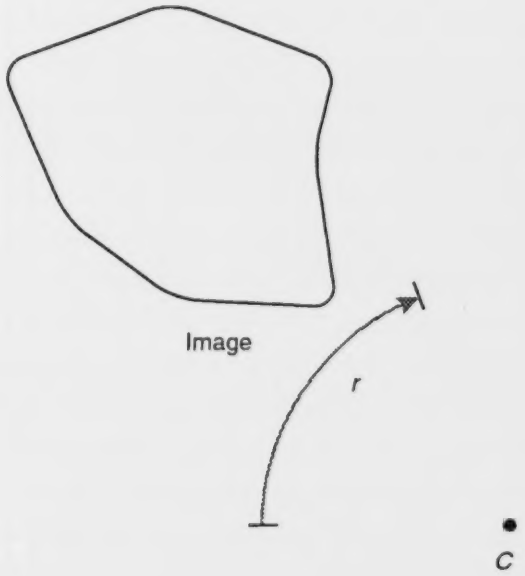
Image

Sample Assessment Problems

11. Below is an image of a figure after it was rotated using arrow *r* and center *c*.

    a. Draw the original.

    b. Could a different arrow be used to get the same original and image? If yes, draw it. If no, explain why not.



12. Reflect figure S using line *h*.

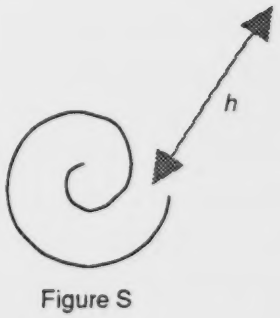
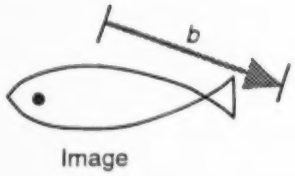


Figure S

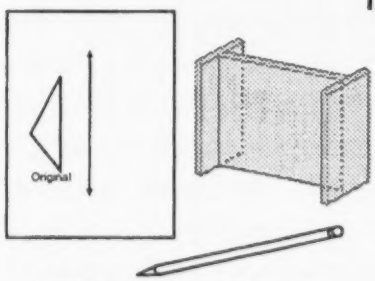
13. Below is the image of a figure *after* it was translated using arrow *b*. Draw the original figure.



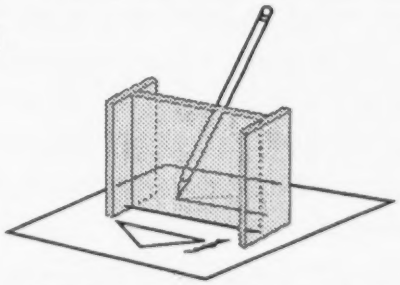
Image

Recording Motions on Paper

Reflection

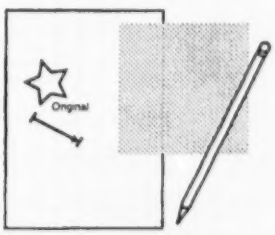


Problem, Mira®, and pencil

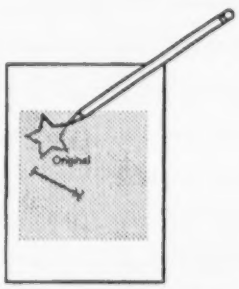


Set Mira® down on the line of reflection and trace the reflection of the original figure. Label the reflection as *image*.

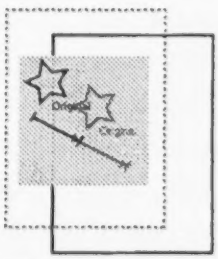
Translation



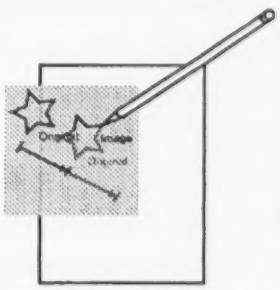
Problem, tracing paper, and pencil



Trace the original figure and the translation arrow. Label the figure *original*.

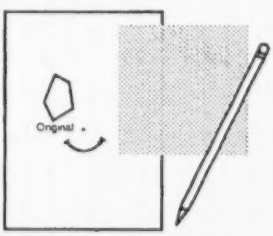


Slide the problem paper (not the tracing paper) in the direction of the arrow until the ends of the arrows match.

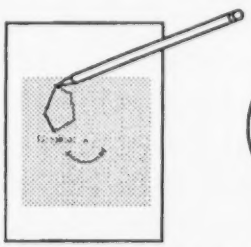


Trace the translated figure and label it *image*.

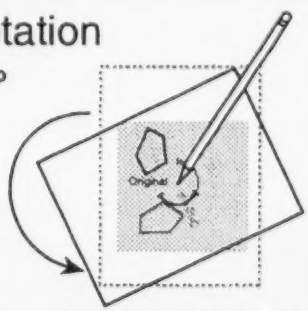
Rotation



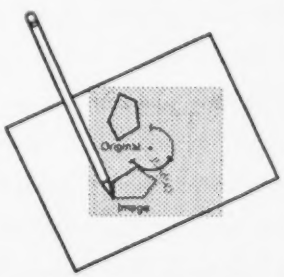
Problem, tracing paper, and pencil



Trace the original figure, the center of rotation, and the rotation arrow. Label the figure *original*.



Use your pencil to keep the center of rotation fixed. Rotate the problem paper (not the tracing paper) in the direction indicated by the rotation arrow until the ends of the arrows match.



Trace the rotated figure and label it *image*.

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ISBN 1-58351-024-9

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